2

Equations and Inequalities
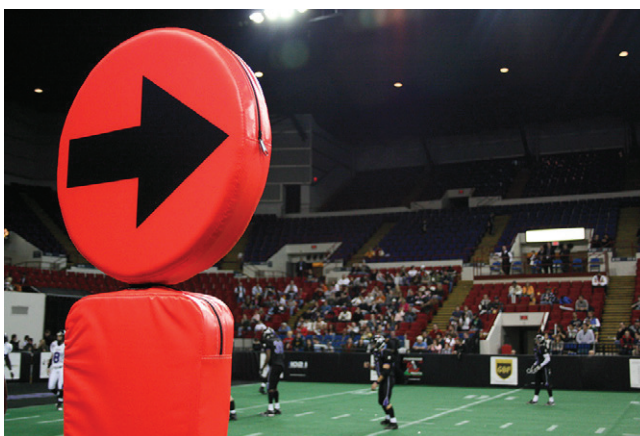

Figure 1

CHAPTER OUTLINE

Introduction

For most people, the term territorial possession indicates restrictions, usually dealing with trespassing or rite of passage and takes place in some foreign location. What most Americans do not realize is that from September through December, territorial possession dominates our lifestyles while watching the NFL. In this area, territorial possession is governed by the referees who make their decisions based on what the chains reveal. If the ball is at point A (x_1, y_1), then it is up to the quarterback to decide which route to point B (x_2, y_2), the end zone, is most feasible.

LEARNING OBJECTIVES

In this section you will:

- Solve quadratic equations by factoring.
- Solve quadratic equations by the square root property.
- Solve quadratic equations by completing the square.
- Solve quadratic equations by using the quadratic formula.

2.5 QUADRATIC EQUATIONS

Figure 1

The computer monitor on the left in **Figure 1** is a 23.6-inch model and the one on the right is a 27-inch model. Proportionally, the monitors appear very similar. If there is a limited amount of space and we desire the largest monitor possible, how do we decide which one to choose? In this section, we will learn how to solve problems such as this using four different methods.

Solving Quadratic Equations by Factoring

An equation containing a second-degree polynomial is called a quadratic equation. For example, equations such as $2x^2 + 3x - 1 = 0$ and $x^2 - 4 = 0$ are quadratic equations. They are used in countless ways in the fields of engineering, architecture, finance, biological science, and, of course, mathematics.

Often the easiest method of solving a quadratic equation is factoring. Factoring means finding expressions that can be multiplied together to give the expression on one side of the equation.

If a quadratic equation can be factored, it is written as a product of linear terms. Solving by factoring depends on the zero-product property, which states that if $a \cdot b = 0$, then $a = 0$ or $b = 0$, where a and b are real numbers or algebraic expressions. In other words, if the product of two numbers or two expressions equals zero, then one of the numbers or one of the expressions must equal zero because zero multiplied by anything equals zero.

Multiplying the factors expands the equation to a string of terms separated by plus or minus signs. So, in that sense, the operation of multiplication undoes the operation of factoring. For example, expand the factored expression $(x - 2)(x + 3)$ by multiplying the two factors together.

$$(x - 2)(x + 3) = x^2 + 3x - 2x - 6$$
$$= x^2 + x - 6$$

The product is a quadratic expression. Set equal to zero, $x^2 + x - 6 = 0$ is a quadratic equation. If we were to factor the equation, we would get back the factors we multiplied.

The process of factoring a quadratic equation depends on the leading coefficient, whether it is 1 or another integer. We will look at both situations; but first, we want to confirm that the equation is written in standard form, $ax^2 + bx + c = 0$, where a, b, and c are real numbers, and $a \neq 0$. The equation $x^2 + x - 6 = 0$ is in standard form.

We can use the zero-product property to solve quadratic equations in which we first have to factor out the greatest common factor (GCF), and for equations that have special factoring formulas as well, such as the difference of squares, both of which we will see later in this section.

> ***the zero-product property and quadratic equations***
> The **zero-product property** states
> $$\text{If } a \cdot b = 0, \text{ then } a = 0 \text{ or } b = 0,$$
> where a and b are real numbers or algebraic expressions.
>
> A **quadratic equation** is an equation containing a second-degree polynomial; for example
> $$ax^2 + bx + c = 0$$
> where a, b, and c are real numbers, and if $a \neq 0$, it is in standard form.

Solving Quadratics with a Leading Coefficient of 1

In the quadratic equation $x^2 + x - 6 = 0$, the leading coefficient, or the coefficient of x^2, is 1. We have one method of factoring quadratic equations in this form.

How To...

Given a quadratic equation with the leading coefficient of 1, factor it.

1. Find two numbers whose product equals c and whose sum equals b.
2. Use those numbers to write two factors of the form $(x + k)$ or $(x - k)$, where k is one of the numbers found in step 1. Use the numbers exactly as they are. In other words, if the two numbers are 1 and -2, the factors are $(x + 1)(x - 2)$.
3. Solve using the zero-product property by setting each factor equal to zero and solving for the variable.

Example 1   **Factoring and Solving a Quadratic with Leading Coefficient of 1**

Factor and solve the equation: $x^2 + x - 6 = 0$.

Solution   To factor $x^2 + x - 6 = 0$, we look for two numbers whose product equals -6 and whose sum equals 1. Begin by looking at the possible factors of -6.
$$1 \cdot (-6)$$
$$(-6) \cdot 1$$
$$2 \cdot (-3)$$
$$3 \cdot (-2)$$

The last pair, $3 \cdot (-2)$ sums to 1, so these are the numbers. Note that only one pair of numbers will work. Then, write the factors.
$$(x - 2)(x + 3) = 0$$

To solve this equation, we use the zero-product property. Set each factor equal to zero and solve.
$$(x - 2)(x + 3) = 0$$
$$(x - 2) = 0$$
$$x = 2$$
$$(x + 3) = 0$$
$$x = -3$$

The two solutions are 2 and -3. We can see how the solutions relate to the graph in **Figure 2**. The solutions are the x-intercepts of $x^2 + x - 6 = 0$.
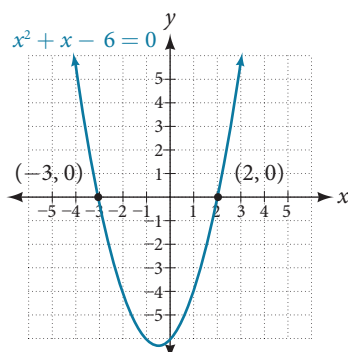
Figure 2

Try It #1

Factor and solve the quadratic equation: $x^2 - 5x - 6 = 0$.

Example 2   Solve the Quadratic Equation by Factoring

Solve the quadratic equation by factoring: $x^2 + 8x + 15 = 0$.

Solution   Find two numbers whose product equals 15 and whose sum equals 8. List the factors of 15.

$$1 \cdot 15$$
$$3 \cdot 5$$
$$(-1) \cdot (-15)$$
$$(-3) \cdot (-5)$$

The numbers that add to 8 are 3 and 5. Then, write the factors, set each factor equal to zero, and solve.

$$(x + 3)(x + 5) = 0$$
$$(x + 3) = 0$$
$$x = -3$$
$$(x + 5) = 0$$
$$x = -5$$

The solutions are -3 and -5.

Try It #2

Solve the quadratic equation by factoring: $x^2 - 4x - 21 = 0$.

Example 3   Using the Zero-Product Property to Solve a Quadratic Equation Written as the Difference of Squares

Solve the difference of squares equation using the zero-product property: $x^2 - 9 = 0$.

Solution   Recognizing that the equation represents the difference of squares, we can write the two factors by taking the square root of each term, using a minus sign as the operator in one factor and a plus sign as the operator in the other. Solve using the zero-factor property.

$$x^2 - 9 = 0$$
$$(x - 3)(x + 3) = 0$$
$$(x - 3) = 0$$
$$x = 3$$
$$(x + 3) = 0$$
$$x = -3$$

The solutions are 3 and -3.

Try It #3

Solve by factoring: $x^2 - 25 = 0$.

Factoring and Solving a Quadratic Equation of Higher Order

When the leading coefficient is not 1, we factor a quadratic equation using the method called grouping, which requires four terms. With the equation in standard form, let's review the grouping procedures:

1. With the quadratic in standard form, $ax^2 + bx + c = 0$, multiply $a \cdot c$.
2. Find two numbers whose product equals ac and whose sum equals b.
3. Rewrite the equation replacing the bx term with two terms using the numbers found in step 1 as coefficients of x.
4. Factor the first two terms and then factor the last two terms. The expressions in parentheses must be exactly the same to use grouping.
5. Factor out the expression in parentheses.
6. Set the expressions equal to zero and solve for the variable.

Example 4   Solving a Quadratic Equation Using Grouping

Use grouping to factor and solve the quadratic equation: $4x^2 + 15x + 9 = 0$.

Solution   First, multiply ac : $4(9) = 36$. Then list the factors of 36.

$$1 \cdot 36$$
$$2 \cdot 18$$
$$3 \cdot 12$$
$$4 \cdot 9$$
$$6 \cdot 6$$

The only pair of factors that sums to 15 is $3 + 12$. Rewrite the equation replacing the b term, $15x$, with two terms using 3 and 12 as coefficients of x. Factor the first two terms, and then factor the last two terms.

$$4x^2 + 3x + 12x + 9 = 0$$
$$x(4x + 3) + 3(4x + 3) = 0$$
$$(4x + 3)(x + 3) = 0$$

Solve using the zero-product property.

$$(4x + 3)(x + 3) = 0$$
$$(4x + 3) = 0$$
$$x = -\frac{3}{4}$$
$$(x + 3) = 0$$
$$x = -3$$

The solutions are $-\dfrac{3}{4}$ and -3. See **Figure 3**.
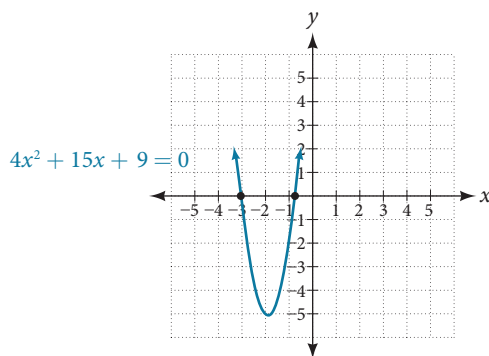

Figure 3

Try It #4

Solve using factoring by grouping: $12x^2 + 11x + 2 = 0$.

Example 5   Solving a Higher Degree Quadratic Equation by Factoring

Solve the equation by factoring: $-3x^3 - 5x^2 - 2x = 0$.

Solution   This equation does not look like a quadratic, as the highest power is 3, not 2. Recall that the first thing we want to do when solving any equation is to factor out the GCF, if one exists. And it does here. We can factor out $-x$ from all of the terms and then proceed with grouping.

$$-3x^3 - 5x^2 - 2x = 0$$
$$-x(3x^2 + 5x + 2) = 0$$

Use grouping on the expression in parentheses.

$$-x(3x^2 + 3x + 2x + 2) = 0$$
$$-x[3x(x + 1) + 2(x + 1)] = 0$$
$$-x(3x + 2)(x + 1) = 0$$

Now, we use the zero-product property. Notice that we have three factors.

$$-x = 0$$
$$x = 0$$
$$3x + 2 = 0$$
$$x = -\frac{2}{3}$$
$$x + 1 = 0$$
$$x = -1$$

The solutions are 0, $-\dfrac{2}{3}$, and -1.

Try It #5

Solve by factoring: $x^3 + 11x^2 + 10x = 0$.

Using the Square Root Property

When there is no linear term in the equation, another method of solving a quadratic equation is by using the **square root property**, in which we isolate the x^2 term and take the square root of the number on the other side of the equals sign. Keep in mind that sometimes we may have to manipulate the equation to isolate the x^2 term so that the square root property can be used.

> **the square root property**
> With the x^2 term isolated, the square root property states that:
> $$\text{if } x^2 = k, \text{ then } x = \pm \sqrt{k}$$
> where k is a nonzero real number.

How To...

Given a quadratic equation with an x^2 term but no x term, use the square root property to solve it.

1. Isolate the x^2 term on one side of the equal sign.
2. Take the square root of both sides of the equation, putting a $\pm$ sign before the expression on the side opposite the squared term.
3. Simplify the numbers on the side with the $\pm$ sign.

Example 6   Solving a Simple Quadratic Equation Using the Square Root Property

Solve the quadratic using the square root property: $x^2 = 8$.

Solution   Take the square root of both sides, and then simplify the radical. Remember to use a $\pm$ sign before the radical symbol.

$$x^2 = 8$$
$$x = \pm \sqrt{8}$$
$$= \pm 2\sqrt{2}$$

The solutions are $2\sqrt{2}$ and $-2\sqrt{2}$.

Example 7   Solving a Quadratic Equation Using the Square Root Property

Solve the quadratic equation: $4x^2 + 1 = 7$.

Solution   First, isolate the x^2 term. Then take the square root of both sides.

$$4x^2 + 1 = 7$$
$$4x^2 = 6$$
$$x^2 = \frac{6}{4}$$
$$x = \pm \frac{\sqrt{6}}{2}$$

The solutions are $\dfrac{\sqrt{6}}{2}$ and $-\dfrac{\sqrt{6}}{2}$.

Try It #6

Solve the quadratic equation using the square root property: $3(x - 4)^2 = 15$.

Completing the Square

Not all quadratic equations can be factored or can be solved in their original form using the square root property. In these cases, we may use a method for solving a quadratic equation known as **completing the square**. Using this method, we add or subtract terms to both sides of the equation until we have a perfect square trinomial on one side of the equal sign. We then apply the square root property. To complete the square, the leading coefficient, a, must equal 1. If it does not, then divide the entire equation by a. Then, we can use the following procedures to solve a quadratic equation by completing the square.

We will use the example $x^2 + 4x + 1 = 0$ to illustrate each step.

1. Given a quadratic equation that cannot be factored, and with $a = 1$, first add or subtract the constant term to the right sign of the equal sign.
$$x^2 + 4x = -1$$

2. Multiply the b term by $\frac{1}{2}$ and square it.
$$\frac{1}{2}(4) = 2$$
$$2^2 = 4$$

3. Add $\left(\frac{1}{2}b\right)^2$ to both sides of the equal sign and simplify the right side. We have
$$x^2 + 4x + 4 = -1 + 4$$
$$x^2 + 4x + 4 = 3$$

4. The left side of the equation can now be factored as a perfect square.
$$x^2 + 4x + 4 = 3$$
$$(x + 2)^2 = 3$$

5. Use the square root property and solve.
$$\sqrt{(x + 2)^2} = \pm\sqrt{3}$$
$$x + 2 = \pm\sqrt{3}$$
$$x = -2 \pm\sqrt{3}$$

6. The solutions are $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$.

Example 8   Solving a Quadratic by Completing the Square

Solve the quadratic equation by completing the square: $x^2 - 3x - 5 = 0$.

Solution   First, move the constant term to the right side of the equal sign.
$$x^2 - 3x = 5$$

Then, take $\frac{1}{2}$ of the b term and square it.
$$\frac{1}{2}(-3) = -\frac{3}{2}$$
$$\left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

Add the result to both sides of the equal sign.
$$x^2 - 3x + \left(-\frac{3}{2}\right)^2 = 5 + \left(-\frac{3}{2}\right)^2$$
$$x^2 - 3x + \frac{9}{4} = 5 + \frac{9}{4}$$

Factor the left side as a perfect square and simplify the right side.
$$\left(x - \frac{3}{2}\right)^2 = \frac{29}{4}$$

Use the square root property and solve.

$$\sqrt{\left(x - \frac{3}{2}\right)^2} = \pm\sqrt{\frac{29}{4}}$$

$$\left(x - \frac{3}{2}\right) = \pm\frac{\sqrt{29}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{29}}{2}$$

The solutions are $\dfrac{3 + \sqrt{29}}{2}$ and $\dfrac{3 - \sqrt{29}}{2}$.

Try It #7

Solve by completing the square: $x^2 - 6x = 13$.

Using the Quadratic Formula

The fourth method of solving a quadratic equation is by using the quadratic formula, a formula that will solve all quadratic equations. Although the quadratic formula works on any quadratic equation in standard form, it is easy to make errors in substituting the values into the formula. Pay close attention when substituting, and use parentheses when inserting a negative number.

We can derive the quadratic formula by completing the square. We will assume that the leading coefficient is positive; if it is negative, we can multiply the equation by -1 and obtain a positive a. Given $ax^2 + bx + c = 0$, $a \neq 0$, we will complete the square as follows:

1. First, move the constant term to the right side of the equal sign:
$$ax^2 + bx = -c$$

2. As we want the leading coefficient to equal 1, divide through by a:
$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

3. Then, find $\frac{1}{2}$ of the middle term, and add $\left(\frac{1b}{2a}\right)^2 = \frac{b^2}{4a^2}$ to both sides of the equal sign:
$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$

4. Next, write the left side as a perfect square. Find the common denominator of the right side and write it as a single fraction:
$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

5. Now, use the square root property, which gives
$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

6. Finally, add $-\dfrac{b}{2a}$ to both sides of the equation and combine the terms on the right side. Thus,
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

---

the quadratic formula

Written in standard form, $ax^2 + bx + c = 0$, any quadratic equation can be solved using the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are real numbers and $a \neq 0$.

Given a quadratic equation, solve it using the quadratic formula

1. Make sure the equation is in standard form: $ax^2 + bx + c = 0$.
2. Make note of the values of the coefficients and constant term, a, b, and c.
3. Carefully substitute the values noted in step 2 into the equation. To avoid needless errors, use parentheses around each number input into the formula.
4. Calculate and solve.

Example 9   Solve the Quadratic Equation Using the Quadratic Formula

Solve the quadratic equation: $x^2 + 5x + 1 = 0$.

Solution   Identify the coefficients: $a = 1$, $b = 5$, $c = 1$. Then use the quadratic formula.

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{-5 \pm \sqrt{25 - 4}}{2}$$

$$= \frac{-5 \pm \sqrt{21}}{2}$$

Example 10   Solving a Quadratic Equation with the Quadratic Formula

Use the quadratic formula to solve $x^2 + x + 2 = 0$.

Solution   First, we identify the coefficients: $a = 1$, $b = 1$, and $c = 2$.

Substitute these values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(1) \pm \sqrt{(1)^2 - (4) \cdot (1) \cdot (2)}}{2 \cdot 1}$$

$$= \frac{-1 \pm \sqrt{1 - 8}}{2}$$

$$= \frac{-1 \pm \sqrt{-7}}{2}$$

$$= \frac{-1 \pm i\sqrt{7}}{2}$$

The solutions to the equation are $\dfrac{-1 + i\sqrt{7}}{2}$ and $\dfrac{-1 - i\sqrt{7}}{2}$.

Try It #8

Solve the quadratic equation using the quadratic formula: $9x^2 + 3x - 2 = 0$.

The Discriminant

The quadratic formula not only generates the solutions to a quadratic equation, it tells us about the nature of the solutions when we consider the discriminant, or the expression under the radical, $b^2 - 4ac$. The discriminant tells us whether the solutions are real numbers or complex numbers, and how many solutions of each type to expect. **Table 1** relates the value of the discriminant to the solutions of a quadratic equation.

Value of Discriminant	Results
$b^2 - 4ac = 0$	One rational solution (double solution)
$b^2 - 4ac > 0$, perfect square	Two rational solutions
$b^2 - 4ac > 0$, not a perfect square	Two irrational solutions
$b^2 - 4ac < 0$	Two complex solutions

Table 1

---

the discriminant

For $ax^2 + bx + c = 0$, where a, b, and c are real numbers, the **discriminant** is the expression under the radical in the quadratic formula: $b^2 - 4ac$. It tells us whether the solutions are real numbers or complex numbers and how many solutions of each type to expect.

Example 11   **Using the Discriminant to Find the Nature of the Solutions to a Quadratic Equation**

Use the discriminant to find the nature of the solutions to the following quadratic equations:

    a. $x^2 + 4x + 4 = 0$ **b.** $8x^2 + 14x + 3 = 0$ **c.** $3x^2 - 5x - 2 = 0$ **d.** $3x^2 - 10x + 15 = 0$

Solution   Calculate the discriminant $b^2 - 4ac$ for each equation and state the expected type of solutions.

    a. $x^2 + 4x + 4 = 0$
    $b^2 - 4ac = (4)^2 - 4(1)(4) = 0$. There will be one rational double solution.

    b. $8x^2 + 14x + 3 = 0$
    $b^2 - 4ac = (14)^2 - 4(8)(3) = 100$. As 100 is a perfect square, there will be two rational solutions.

    c. $3x^2 - 5x - 2 = 0$
    $b^2 - 4ac = (-5)^2 - 4(3)(-2) = 49$. As 49 is a perfect square, there will be two rational solutions.

    d. $3x^2 - 10x + 15 = 0$
    $b^2 - 4ac = (-10)^2 - 4(3)(15) = -80$. There will be two complex solutions.

Using the Pythagorean Theorem

One of the most famous formulas in mathematics is the **Pythagorean Theorem**. It is based on a right triangle, and states the relationship among the lengths of the sides as $a^2 + b^2 = c^2$, where a and b refer to the legs of a right triangle adjacent to the 90° angle, and c refers to the hypotenuse. It has immeasurable uses in architecture, engineering, the sciences, geometry, trigonometry, and algebra, and in everyday applications.

We use the Pythagorean Theorem to solve for the length of one side of a triangle when we have the lengths of the other two. Because each of the terms is squared in the theorem, when we are solving for a side of a triangle, we have a quadratic equation. We can use the methods for solving quadratic equations that we learned in this section to solve for the missing side.

The Pythagorean Theorem is given as

$$a^2 + b^2 = c^2$$

where a and b refer to the legs of a right triangle adjacent to the 90° angle, and c refers to the hypotenuse, as shown in **Figure 4**.
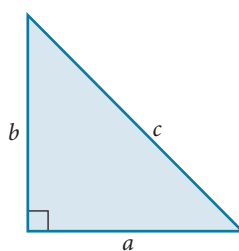

Figure 4

Example 12   **Finding the Length of the Missing Side of a Right Triangle**

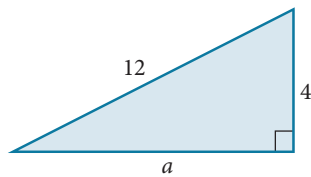Find the length of the missing side of the right triangle in **Figure 5**.

Figure 5

Solution   As we have measurements for side b and the hypotenuse, the missing side is a.

$$a^2 + b^2 = c^2$$
$$a^2 + (4)^2 = (12)^2$$
$$a^2 + 16 = 144$$
$$a^2 = 128$$
$$a = \sqrt{128}$$
$$= 8\sqrt{2}$$

Try It #9

Use the Pythagorean Theorem to solve the right triangle problem: Leg a measures 4 units, leg b measures 3 units. Find the length of the hypotenuse.

Access these online resources for additional instruction and practice with quadratic equations.

- Solving Quadratic Equations by Factoring (http://openstaxcollege.org/l/quadreqfactor)
- The Zero-Product Property (http://openstaxcollege.org/l/zeroprodprop)
- Completing the Square (http://openstaxcollege.org/l/complthesqr)
- Quadratic Formula with Two Rational Solutions (http://openstaxcollege.org/l/quadrformrat)
- Length of a Leg of a Right Triangle (http://openstaxcollege.org/l/leglengthtri)

2.5 SECTION EXERCISES

VERBAL

1. How do we recognize when an equation is quadratic?

2. When we solve a quadratic equation, how many solutions should we always start out seeking? Explain why when solving a quadratic equation in the form $ax^2 + bx + c = 0$ we may graph the equation $y = ax^2 + bx + c$ and have no zeroes (x-intercepts).

3. When we solve a quadratic equation by factoring, why do we move all terms to one side, having zero on the other side?

4. In the quadratic formula, what is the name of the expression under the radical sign $b^2 - 4ac$, and how does it determine the number of and nature of our solutions?

5. Describe two scenarios where using the square root property to solve a quadratic equation would be the most efficient method.

ALGEBRAIC

For the following exercises, solve the quadratic equation by factoring.

6. $x^2 + 4x - 21 = 0$
7. $x^2 - 9x + 18 = 0$
8. $2x^2 + 9x - 5 = 0$
9. $6x^2 + 17x + 5 = 0$

10. $4x^2 - 12x + 8 = 0$
11. $3x^2 - 75 = 0$
12. $8x^2 + 6x - 9 = 0$
13. $4x^2 = 9$

14. $2x^2 + 14x = 36$
15. $5x^2 = 5x + 30$
16. $4x^2 = 5x$
17. $7x^2 + 3x = 0$

18. $\dfrac{x}{3} - \dfrac{9}{x} = 2$

For the following exercises, solve the quadratic equation by using the square root property.

19. $x^2 = 36$
20. $x^2 = 49$
21. $(x - 1)^2 = 25$
22. $(x - 3)^2 = 7$

23. $(2x + 1)^2 = 9$
24. $(x - 5)^2 = 4$

For the following exercises, solve the quadratic equation by completing the square. Show each step.

25. $x^2 - 9x - 22 = 0$
26. $2x^2 - 8x - 5 = 0$
27. $x^2 - 6x = 13$
28. $x^2 + \dfrac{2}{3}x - \dfrac{1}{3} = 0$

29. $2 + z = 6z^2$
30. $6p^2 + 7p - 20 = 0$
31. $2x^2 - 3x - 1 = 0$

For the following exercises, determine the discriminant, and then state how many solutions there are and the nature of the solutions. Do not solve.

32. $2x^2 - 6x + 7 = 0$
33. $x^2 + 4x + 7 = 0$
34. $3x^2 + 5x - 8 = 0$
35. $9x^2 - 30x + 25 = 0$

36. $2x^2 - 3x - 7 = 0$
37. $6x^2 - x - 2 = 0$

For the following exercises, solve the quadratic equation by using the quadratic formula. If the solutions are not real, state *No Real Solution*.

38. $2x^2 + 5x + 3 = 0$
39. $x^2 + x = 4$
40. $2x^2 - 8x - 5 = 0$
41. $3x^2 - 5x + 1 = 0$

42. $x^2 + 4x + 2 = 0$
43. $4 + \dfrac{1}{x} - \dfrac{1}{x^2} = 0$

TECHNOLOGY

For the following exercises, enter the expressions into your graphing utility and find the zeroes to the equation (the x-intercepts) by using **2nd CALC 2:zero**. Recall finding zeroes will ask left bound (move your cursor to the left of the zero, enter), then right bound (move your cursor to the right of the zero, enter), then guess (move your cursor between the bounds near the zero, enter). Round your answers to the nearest thousandth.

44. $Y_1 = 4x^2 + 3x - 2$
45. $Y_1 = -3x^2 + 8x - 1$
46. $Y_1 = 0.5x^2 + x - 7$

47. To solve the quadratic equation $x^2 + 5x - 7 = 4$, we can graph these two equations

$$Y_1 = x^2 + 5x - 7 \qquad\qquad Y_2 = 4$$

and find the points of intersection. Recall **2nd CALC 5:intersection**. Do this and find the solutions to the nearest tenth.

48. To solve the quadratic equation $0.3x^2 + 2x - 4 = 2$, we can graph these two equations

$$Y_1 = 0.3x^2 + 2x - 4 \qquad\qquad Y_2 = 2$$

and find the points of intersewction. Recall **2nd CALC 5:intersection**. Do this and find the solutions to the nearest tenth.

EXTENSIONS

49. Beginning with the general form of a quadratic equation, $ax^2 + bx + c = 0$, solve for x by using the completing the square method, thus deriving the quadratic formula.

50. Show that the sum of the two solutions to the quadratic equation is $-\dfrac{b}{a}$.

51. A person has a garden that has a length 10 feet longer than the width. Set up a quadratic equation to find the dimensions of the garden if its area is 119 ft.^{2}. Solve the quadratic equation to find the length and width.

52. Abercrombie and Fitch stock had a price given as $P = 0.2t^2 - 5.6t + 50.2$, where t is the time in months from 1999 to 2001. ($t = 1$ is January 1999). Find the two months in which the price of the stock was $30.

53. Suppose that an equation is given $p = -2x^2 + 280x - 1000$, where x represents the number of items sold at an auction and p is the profit made by the business that ran the auction. How many items sold would make this profit a maximum? Solve this by graphing the expression in your graphing utility and finding the maximum using **2nd CALC maximum**. To obtain a good window for the curve, set x [0,200] and y [0,10000].

REAL-WORLD APPLICATIONS

54. A formula for the normal systolic blood pressure for a man age A, measured in mmHg, is given as $P = 0.006A^2 - 0.02A + 120$. Find the age to the nearest year of a man whose normal blood pressure measures 125 mmHg.

55. The cost function for a certain company is $C = 60x + 300$ and the revenue is given by $R = 100x - 0.5x^2$. Recall that profit is revenue minus cost. Set up a quadratic equation and find two values of x (production level) that will create a profit of $300.

56. A falling object travels a distance given by the formula $d = 5t + 16t^2$ ft, where t is measured in seconds. How long will it take for the object to traveled 74 ft?

57. A vacant lot is being converted into a community garden. The garden and the walkway around its perimeter have an area of 378 ft^2. Find the width of the walkway if the garden is 12 ft. wide by 15 ft. long.
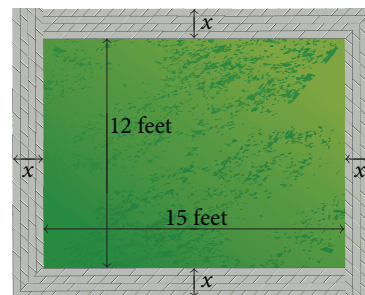
58. An epidemiological study of the spread of a certain influenza strain that hit a small school population found that the total number of students, P, who contracted the flu t days after it broke out is given by the model $P = -t^2 + 13t + 130$, where $1 \leq t \leq 6$. Find the day that 160 students had the flu. Recall that the restriction on t is at most 6.