GUIDED NOTES – 3.5 TRANSFORMATIONS OF FUNCTIONS

LEARNING OBJECTIVES

In this section, you will:

- Graph functions using vertical and horizontal shifts.
- Graph functions using reflections about the x-axis and the y-axis.
- Determine whether a function is even, odd, or neither from its graph.
- Graph functions using compressions and stretches.
- Combine transformations.

GRAPHING FUNCTIONS USING VERTICAL AND HORIZONTAL SHIFTS

Study the box in your textbook section titled "vertical shift."

- Given a function $f(x)$, a new function $g(x) =$ _____, where k is a _____, is a vertical shift of the function $f(x)$. All the outputs will change by _____ units.
    - If k is positive, _____.
    - If k is negative, _____.

- Write out the 3 step procedure for creating a new row to represent a vertical shift, given a tabular function

    1.

    2.

    3.

Try It: Read Examples 1 and 2 in the text, then answer the following.

The function $h(t) = -4.9t^2 + 30t$ gives the height h of a ball (in meters) thrown upward from the ground after t seconds. Suppose the ball was instead thrown from the top of a 10-m building. Relate this new height function $b(t)$ to $h(t)$, and then find a formula for $b(t)$.

Study the box in your textbook section titled "horizontal shift."

- Given a function $f(x)$, a new function $g(x) =$ _____, where h is a _____, is a horizontal shift of the function f.
    - If h is positive, _____.
    - If h is negative, _____.

- Write out the 3 step procedure for creating a new row to represent a horizontal shift, given a tabular function.

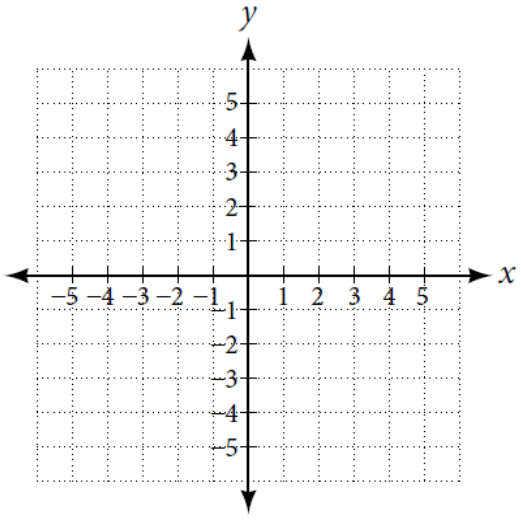
    1.

    2.

    3.

Try It: Read Examples 5 and 6 in the text, then answer the following.

Given the function $f(x) = \sqrt{x}$, graph the original function $f(x)$ and the transformation $g(x) = f(x + 2)$ on the same axes. Is this a horizontal or vertical shift? Which way was the graph shifted and by how many units?



- Write out the 4 step procedure for sketching a graph, given a function and both a vertical and horizontal shift.

    1.

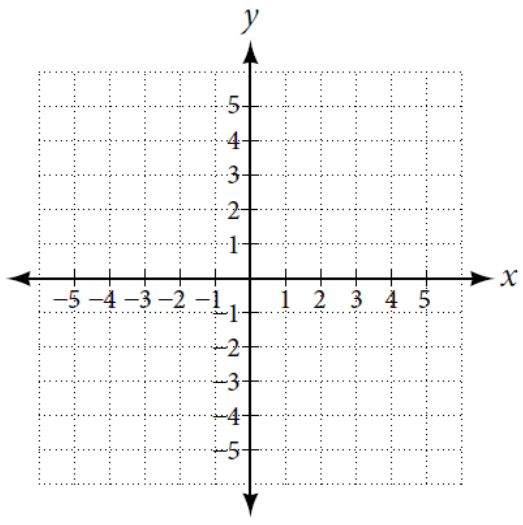
    2.

    3.

    4.

Try It: Read Example 7 in the text, then answer the following.

Given $f(x) = |x|$, sketch a graph of $h(x) = f(x - 2) + 4$.



Try It: Read Example 8 in the text, then answer the following.

Write a formula for a transformation of the toolkit reciprocal function $f(x) = \frac{1}{x}$ that shifts the function's graph one unit to the right and one unit up.

GRAPHING FUNCTIONS USING REFLECTIONS ABOUT THE AXES

Study the box in your textbook section titled "reflections."

- Given a function $f(x)$, a new function $g(x) =$ _____, is a vertical reflection of the function $f(x)$, sometimes called a _____.
- Given a function $f(x)$, a new function $g(x) =$ _____, is a horizontal reflection of the function $f(x)$, sometimes called a _____.

- Write out the 2 step procedure for reflecting a graph both vertically and horizontally, given a function.

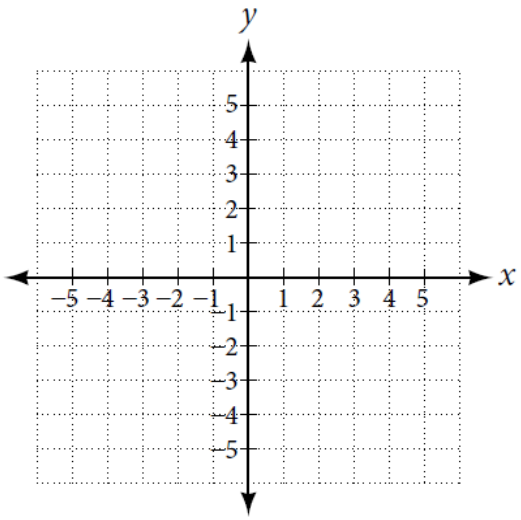
  1.

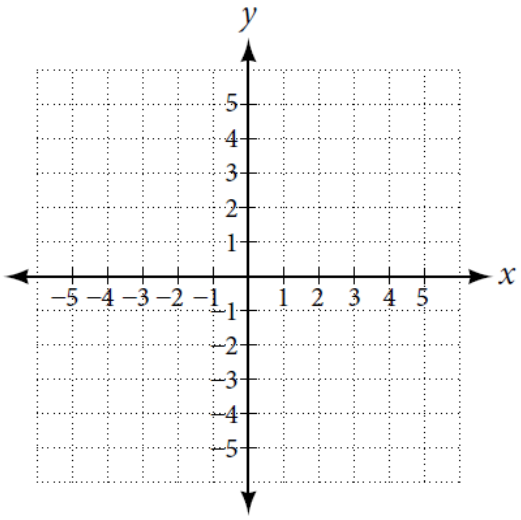
  2.

Try It: Read Example 9 in the text, then answer the following.

Reflect the graph of $f(x) = |x + 1|$

a. vertically **b.** horizontally





Try It: Read Example 10 in the text, then answer the following.

A function $f(x)$ is given as **Table 9**. Create a table for the functions below.

x	-2	0	2	4
$f(x)$	5	10	15	20

Table 9

a. $g(x) = -f(x)$

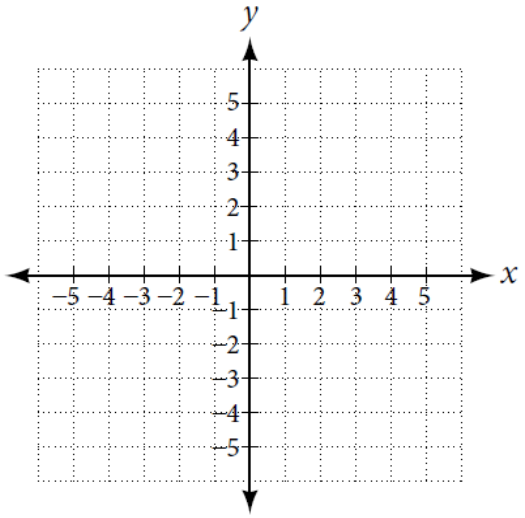
x				
$g(x)$				

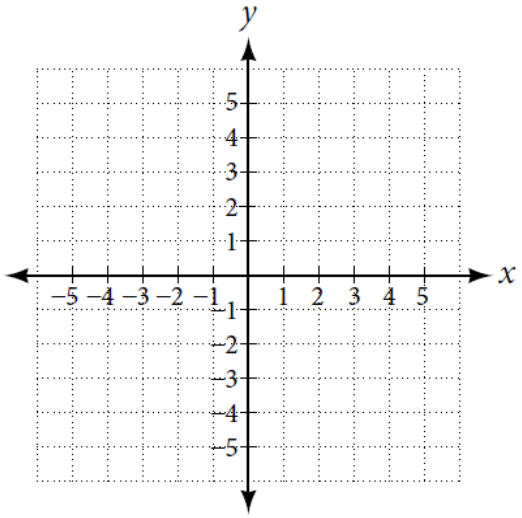
b. $g(x) = f(-x)$

x				
$g(x)$				

Try It: Read Example 11 in the text, then answer the following.

Given the toolkit function $f(x) = x^2$, graph $g(x) = -f(x)$ and $h(x) = f(-x)$. Take note of any surprising behavior for these functions.





DETERMINING EVEN AND ODD FUNCTIONS

Study the box in your textbook section titled "even and odd functions."

- A function is called an even function if for every input x: _____.
    - The graph of an even function is symmetrical about the _____.
- A function is called an odd function if for every input x: _____.
    - The graph of an odd function is symmetrical about the _____.

Try It: Read Example 12 in the text, then answer the following.

Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd, or neither?

GRAPHING FUNCTIONS USING STRETCHES AND COMPRESSIONS

Study the box in your textbook section titled "vertical stretches and compressions."

- Given a function $f(x)$, a new function $g(x) =$ _____, were a is constant, is a vertical stretch or vertical compression of $f(x)$.
    - The graph will be stretched when _____.

- The graph will be compressed when _____.
- The graph will have a combination of vertical stretch or compression with a vertical reflection when _____.

- Write out the 3 step procedure for graphing a vertical stretch, given a function.

1.

2.

3.

- Write out the 2 step procedure for creating a table for a vertical compression, given a tabular function and assuming that the transformation is a vertical stretch or compression.

1.

2.

Try It: Read Example 14 in the text, then answer the following.

A function f is given as **Table 12**. Create a table for the function $g(x) = \frac{3}{4}f(x)$.

x	2	4	6	8
$f(x)$	12	16	20	0

Table 12

x				
$g(x)$				

Try It: Read Example 15 in the text, then answer the following.

Write the formula for the function that we get when we stretch the identity toolkit function by a factor of 3, and then shift it down by 2 units.

Study the box in your textbook section titled "horizontal stretches and compressions."

- Given a function $f(x)$, a new function $g(x) =$ _____, where b is constant, is a horizontal stretch or horizontal compression of $f(x)$.
    - The graph will be compressed by $\frac{1}{b}$ when _____.
    - The graph will be stretched by $\frac{1}{b}$ when _____.
    - The graph will have a combination of horizontal stretch or compression with a horizontal reflection when _____.

- Write out the 2 step procedure for sketching a horizontal compression or stretch, given a description of a function.

    1.

    2.

Try It: Read Examples 16, 17, and 18 in the text, then answer the following.

Write a formula for the toolkit square root function horizontally stretched by a factor of 3.

PERFORMING A SEQUENCE OF TRANSFORMATIONS

Study the box in your textbook section titled "combining transformations."

- When combining vertical transformations written in the form $af(x) + k$, first _____,

  and then _____.

- When combining horizontal transformations written in the form $f(bx - h)$, first _____,

  and then _____.

- When combining horizontal transformations written in the form $f(b(x - h))$, first

  _____, and then _____.

  ** Remember that horizontal and vertical transformations are independent, so it does not matter whether horizontal or vertical transformations are performed first.*