GUIDED NOTES – 3.5 TRANSFORMATIONS OF FUNCTIONS

LEARNING OBJECTIVES

In this section, you will:

- Graph functions using vertical and horizontal shifts.
- Graph functions using reflections about the x-axis and the y-axis.
- Determine whether a function is even, odd, or neither from its graph.
- Graph functions using compressions and stretches.
- Combine transformations.

GRAPHING FUNCTIONS USING VERTICAL AND HORIZONTAL SHIFTS

*Study the box in your textbook section titled “vertical shift.”*

- Given a function \( f(x) \), a new function \( g(x) = \ldots \), where \( k \) is a \( \ldots \), is a vertical shift of the function \( f(x) \). All the outputs will change by \( \ldots \) units.
  - If \( k \) is positive, \( \ldots \).
  - If \( k \) is negative, \( \ldots \).

- Write out the 3 step procedure for creating a new row to represent a vertical shift, given a tabular function
  1.
  2.
  3.

*Try It:* Read Examples 1 and 2 in the text, then answer the following.

The function \( h(t) = -4.9t^2 + 30t \) gives the height \( h \) of a ball (in meters) thrown upward from the ground after \( t \) seconds. Suppose the ball was instead thrown from the top of a 10-m building. Relate this new height function \( b(t) \) to \( h(t) \), and then find a formula for \( b(t) \).
Study the box in your textbook section titled “horizontal shift.”

- Given a function $f(x)$, a new function $g(x) = \underline{\phantom{\text{a function}}} = \underline{\phantom{\text{a function}}}$, where $h$ is a $\underline{\phantom{\text{a number}}}$, is a horizontal shift of the function $f$.
  - If $h$ is positive, $\underline{\phantom{\text{the graph is shifted right by}}}$.
  - If $h$ is negative, $\underline{\phantom{\text{the graph is shifted left by}}}$.

- Write out the 3 step procedure for creating a new row to represent a horizontal shift, given a tabular function.
  1. $\underline{\phantom{\text{Step 1}}}$
  2. $\underline{\phantom{\text{Step 2}}}$
  3. $\underline{\phantom{\text{Step 3}}}$

**Try It:** Read Examples 5 and 6 in the text, then answer the following.

Given the function $f(x) = \sqrt{x}$, graph the original function $f(x)$ and the transformation $g(x) = f(x + 2)$ on the same axes. Is this a horizontal or vertical shift? Which way was the graph shifted and by how many units?
• Write out the 4 step procedure for sketching a graph, given a function and both a vertical and horizontal shift.

1.

2.

3.

4.

Try It: Read Example 7 in the text, then answer the following.

Given \( f(x) = |x| \), sketch a graph of \( h(x) = f(x - 2) + 4 \).

Try It: Read Example 8 in the text, then answer the following.
Write a formula for a transformation of the toolkit reciprocal function \( f(x) = \frac{1}{x} \) that shifts the function’s graph one unit to the right and one unit up.

**GRAPHING FUNCTIONS USING REFLECTIONS ABOUT THE AXES**

*Study the box in your textbook section titled “reflections.”*

- Given a function \( f(x) \), a new function \( g(x) = \) ____________, is a vertical reflection of the function \( f(x) \), sometimes called a ________________.

- Given a function \( f(x) \), a new function \( g(x) = \) ____________, is a horizontal reflection of the function \( f(x) \), sometimes called a ________________.

- Write out the 2 step procedure for reflecting a graph both vertically and horizontally, given a function.

  1. 
  2. 

*Try It:* Read Example 9 in the text, then answer the following.

Reflect the graph of \( f(x) = |x + 1| \)

- a. vertically
- b. horizontally
Try It: Read Example 10 in the text, then answer the following.

A function \( f(x) \) is given as Table 9. Create a table for the functions below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(0)</th>
<th>(2)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 9

a. \( g(x) = -f(x) \)

Try It: Read Example 11 in the text, then answer the following.

Given the toolkit function \( f(x) = x^2 \), graph \( g(x) = -f(x) \) and \( h(x) = f(-x) \). Take note of any surprising behavior for these functions.
**DETERMINING EVEN AND ODD FUNCTIONS**

*Study the box in your textbook section titled “even and odd functions.”*

- A function is called an even function if for every input $x$: ________________.
  - The graph of an even function is symmetrical about the ________________.
- A function is called an odd function if for every input $x$: ________________.
  - The graph of an odd function is symmetrical about the ________________.

**Try It:** Read Example 12 in the text, then answer the following.

Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd, or neither?

**GRAPHING FUNCTIONS USING STRETCHES AND COMPRESSIONS**

*Study the box in your textbook section titled “vertical stretches and compressions.”*

- Given a function $f(x)$, a new function $g(x) = __________$, where $a$ is constant, is a vertical stretch or vertical compression of $f(x)$.
  - The graph will be stretched when ________________.
- The graph will be compressed when ________________.
- The graph will have a combination of vertical stretch or compression with a vertical reflection when ________________.

- Write out the 3 step procedure for graphing a vertical stretch, given a function.

  1.

  2.

  3.

- Write out the 2 step procedure for creating a table for a vertical compression, given a tabular function and assuming that the transformation is a vertical stretch or compression.

  1.

  2.

**Try It:** Read Example 14 in the text, then answer the following.

A function \( f \) is given as **Table 12**. Create a table for the function \( g(x) = \frac{3}{4}f(x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>12</td>
<td>16</td>
<td>20</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 12**

<table>
<thead>
<tr>
<th>( x )</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Try It:** Read Example 15 in the text, then answer the following.
Write the formula for the function that we get when we stretch the identity toolkit function by a factor of 3, and then shift it down by 2 units.

Study the box in your textbook section titled “horizontal stretches and compressions.”

• Given a function \( f(x) \), a new function \( g(x) = \) \( \frac{1}{b} \), where \( b \) is constant, is a horizontal stretch or horizontal compression of \( f(x) \).
  - The graph will be compressed by \( \frac{1}{b} \) when ________.
  - The graph will be stretched by \( \frac{1}{b} \) when ________.
  - The graph will have a combination of horizontal stretch or compression with a horizontal reflection when ________.

• Write out the 2 step procedure for sketching a horizontal compression or stretch, given a description of a function.

  1.

  2.

Try It: Read Examples 16, 17, and 18 in the text, then answer the following.

  Write a formula for the toolkit square root function horizontally stretched by a factor of 3.

PERFORMING A SEQUENCE OF TRANSFORMATIONS
Study the box in your textbook section titled “combining transformations.”

- When combining vertical transformations written in the form $af(x) + k$, first ____________________, and then ____________________.

- When combining horizontal transformations written in the form $f(bx - h)$, first ____________________, and then ____________________.

- When combining horizontal transformations written in the form $f(b(x - h))$, first ____________________, and then ____________________.

* Remember that horizontal and vertical transformations are independent, so it does not matter whether horizontal or vertical transformations are performed first.