GUIDED NOTES – 3.4 COMPOSITION OF FUNCTIONS

LEARNING OBJECTIVES

In this section, you will:

- Combine functions using algebraic operations.
- Create a new function by composition of functions.
- Evaluate composite functions.
- Find the domain of a composite function.
- Decompose a composite function into its component functions.

COMBINING FUNCTIONS USING ALGEBRAIC OPERATIONS

- For two functions $f(x)$ and $g(x)$ with real number outputs, we define new functions $f + g, f - g, fg$, and $\frac{f}{g}$ by the relations:

    - $(f + g)(x) = $ _____

    - $(f - g)(x) = $ _____

    - $(fg)(x) = $ _____

    - $\left(\frac{f}{g}\right)(x) = $ _____, where $g(x) \neq 0$

Try It: Read Example 1 in the text, then answer the following.

Find and simplify the functions $(fg)(x)$ and $(f - g)(x)$.

$$f(x) = x - 1 \text{ and } g(x) = x^2 - 1$$

Are they the same function?

Homework: *You should now be ready to attempt problems 1-3 in "Homework – Section 3.4" on WeBWorK.*

CREATE A FUNCTION BY COMPOSITION OF FUNCTIONS

Study the box in your textbook section titled "composition of functions."

- When the output of one function is used as the input of another, we call the entire operation a

  _____. For any input x and functions f and g, this action defines a composite

  function, which we write as $f \circ g$ such that _____.

* Remember that the product of fg is NOT the same as the function composition $f(g(x))$, because, in general,
$(fg)(x) \neq f(g(x))$.

    Try It: Read Example 2 in the text, then answer the following.

        Using the functions provided, find $f\big(g(x)\big)$ and $g\big(f(x)\big)$. Determine whether the composition of the

        functions is *commutative*.

$$f(x) = x^2 + 3 \qquad g(x) = \frac{1}{x}$$

Homework: *You should now be ready to attempt problem 7 in "Homework – Section 3.3" on WeBWorK.*

EVALUATING COMPOSITE FUNCTIONS

    Try It: Read Example 5 in the text, then answer the following.

        Using **Table 1**, evaluate $f(g(1))$ and $g(f(4))$.

x	$f(x)$	$g(x)$
1	6	3
2	8	5
3	3	2
4	1	7

Table 1

- Write out the 4-step process for evaluating composite functions, given a composite function and graphs of its individual functions.

    1.

    2.

    3.

    4.

Try It: Read Example 6 in the text, then answer the following.
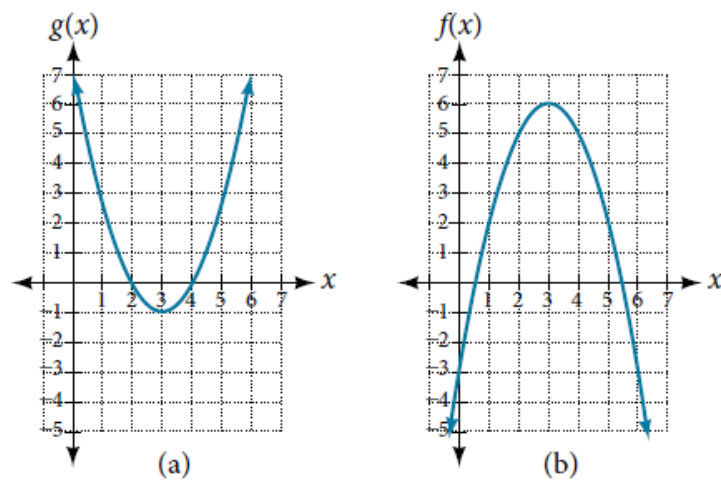
    Using **Figure 1**, evaluate $g(f(2))$.

Figure 1

- Write out the 2-step process for evaluating a function, given a formula for a composite function.

    1.

    2.

Try It: Read Example 7 in the text, then answer the following.

Given $f(t) = t^2 - t$ and $h(x) = 3x + 2$, evaluate

    a. $h(f(2))$

    b. $h(f(-2))$

Homework: *You should now be ready to attempt problems 4-6 in "Homework – Section 3.3" on WeBWorK.*

FINDING THE DOMAIN OF A COMPOSITE FUNCTION

- Write out the 3-step process for determining the domain, given a function composition $f(g(x))$.

    1.

    2.

    3.

Try It: Read Examples 8 and 9 in the text, then answer the following.

Find the domain of $(f \circ g)(x)$ where $f(x) = \dfrac{1}{x-2}$ and $g(x) = \sqrt{x + 4}$

Homework: *You should now be ready to attempt problem 11 in "Homework – Section 3.3" on WeBWorK.*

DECOMPOSING A COMPOSITE FUNCTION INTO ITS COMPONENT FUNCTIONS

Try It: Read Example 10 in the text, then answer the following.

Write $f(x) = \dfrac{4}{3 - \sqrt{4 + x^2}}$ as the composition of two functions.

Homework: *You should now be ready to attempt problems 12-13 in "Homework – Section 3.3" on WeBWorK.*