

## GUIDED NOTES – 5.6 RATIONAL FUNCTIONS

### LEARNING OBJECTIVES

In this section, you will:

- Use arrow notation.
- Solve applied problems involving rational functions.
- Find the domains of rational functions.
- Identify vertical asymptotes.
- Identify horizontal asymptotes.
- Graph rational functions.

### USING ARROW NOTATION

Study Table 1 in your textbook section to complete the table below.

<i>Symbol</i>	<i>Meaning</i>
$x \rightarrow a^+$	$x$ approaches $a$ from the _____ ( $x$ _____ $a$ but close to $a$ )
$x \rightarrow a^-$	$x$ approaches $a$ from the _____ ( $x$ _____ $a$ but close to $a$ )
$x \rightarrow \infty$	$x$ approaches _____ ( $x$ _____ without bound)
$x \rightarrow -\infty$	$x$ approaches _____ ( $x$ _____ without bound)
$f(x) \rightarrow \infty$	The output approaches _____ (The output _____ without bound)
$f(x) \rightarrow -\infty$	The output approaches _____ (The output _____ without bound)
$f(x) \rightarrow a$	The output approaches _____

*Study the box in your textbook section titled “vertical asymptote.”*

- A \_\_\_\_\_ asymptote of a graph is a vertical line  $x = \underline{\hspace{2cm}}$  where the graph tends towards positive or negative infinity as the inputs approach  $a$ .
- How would you write this using arrow notation?

*Study the box in your textbook section titled “horizontal asymptote.”*

- A \_\_\_\_\_ asymptote of a graph is a horizontal line  $y = \underline{\hspace{2cm}}$  where the graph approaches the line as the inputs increase or decrease without bound.
- How would you write this using arrow notation?

- A rational function is a function that can be written as the \_\_\_\_\_ of two polynomial functions.

### **FINDING THE DOMAINS OF RATIONAL FUNCTIONS**

*Study the box in your textbook section titled “domain of a rational function.”*

- The domain of a rational function includes all real numbers except those that cause the denominator to equal \_\_\_\_\_.
- Write out the 3 step process for finding the domain, given a rational function.
  - 1.
  - 2.
  - 3.

**Try It:** Read Example 4 in the text, then answer the following.

Find the domain of  $f(x) = \frac{4x}{5(x-1)(x-5)}$ .

**IDENTIFYING VERTICAL ASYMPTOTES OF RATIONAL FUNCTIONS**

- Write out the 5 step process for identifying any vertical asymptotes of the graph, given a rational function.
  - 1.
  - 2.
  - 3.
  - 4.
  - 5.

*Study the box in your textbook section titled “removable discontinuities of rational functions.”*

- When does a *removable discontinuity* occur?

**Try It:** Read Examples 5 and 6 in the text, then answer the following.

Find the vertical asymptote and removable discontinuities of the graph  $f(x) = \frac{x^2-25}{x^3-6x^2+5x}$ .

## IDENTIFYING HORIZONTAL ASYMPTOTES OF RATIONAL FUNCTIONS

Study the box in your textbook section titled “horizontal asymptotes of rational functions.”

- Degree of numerator is \_\_\_\_\_ degree of denominator, then there is a horizontal asymptote at \_\_\_\_\_.
- Degree of numerator is \_\_\_\_\_ degree of denominator by one, then there is no horizontal asymptote, but there is a \_\_\_\_\_ asymptote.
- Degree of numerator is \_\_\_\_\_ degree of denominator, then there is a horizontal asymptote at \_\_\_\_\_.

**Try It:** Read Examples 7, 8, and 9 in the text, then answer the following.

Find the vertical and horizontal asymptotes of the function  $f(x) = \frac{(2x-1)(2x+1)}{(x-2)(x+3)}$ .

**Homework:** You should now be ready to attempt problems 1-4 in “Homework – Section 5.6” on WeBWorK.

Study the box in your textbook section titled “intercepts of rational functions.”

- A rational function will have a y-intercept when the \_\_\_\_\_ is zero, if the function is defined at zero.
- When will a rational function not have a y-intercept?

- A rational function will have  $x$ -intercepts at the inputs that cause the output to be \_\_\_\_\_. Remember,  $x$ -intercepts can only occur when the \_\_\_\_\_ of the rational function equals zero.

**Homework:** *You should now be ready to attempt problem 5 in “Homework – Section 5.6” on WeBWork.*

### GRAPHING RATIONAL FUNCTIONS

- Write out the 8 step process for sketching a graph, given a rational function.

1.

2.

3.

4.

5.

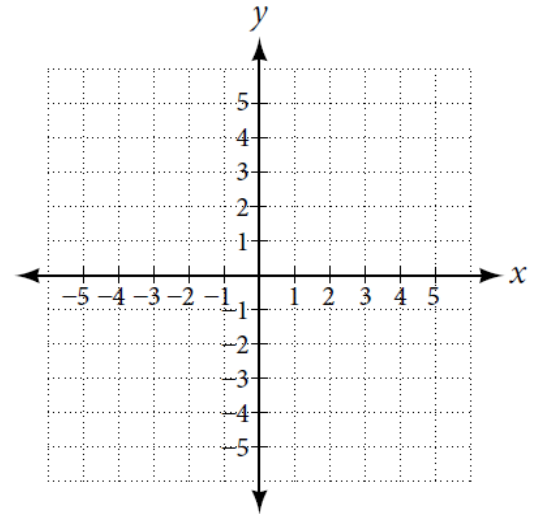
6.

7.

8.

**Try It:** Read Example 11 in the text, then answer the following.

Given the function  $f(x) = \frac{(x+2)^2(x-2)}{2(x-1)^2(x-3)}$ , use the characteristics of polynomials and rational functions to describe its behavior and sketch the function.



### WRITING RATIONAL FUNCTIONS

- Write out the 3 step process for writing a rational equation, given a graph of a rational function.

1.

2.

3.