4

Exponential and Logarithmic Functions

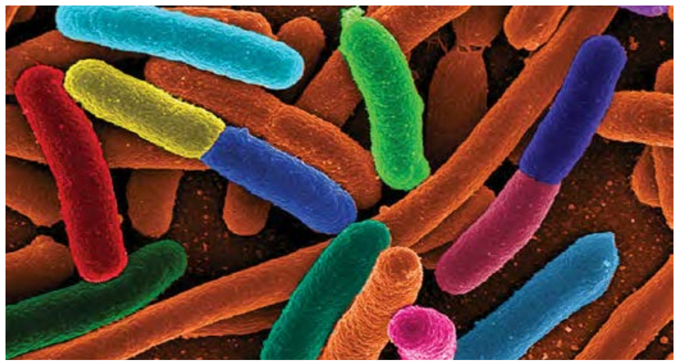


Figure 1 Electron micrograph of *E. Coli* bacteria (credit: "Mattosaurus," Wikimedia Commons)

CHAPTER OUTLINE

Introduction

Focus in on a square centimeter of your skin. Look closer. Closer still. If you could look closely enough, you would see hundreds of thousands of microscopic organisms. They are bacteria, and they are not only on your skin, but in your mouth, nose, and even your intestines. In fact, the bacterial cells in your body at any given moment outnumber your own cells. But that is no reason to feel bad about yourself. While some bacteria can cause illness, many are healthy and even essential to the body.

Bacteria commonly reproduce through a process called binary fission, during which one bacterial cell splits into two. When conditions are right, bacteria can reproduce very quickly. Unlike humans and other complex organisms, the time required to form a new generation of bacteria is often a matter of minutes or hours, as opposed to days or years.[16]

For simplicity's sake, suppose we begin with a culture of one bacterial cell that can divide every hour. **Table 1** shows the number of bacterial cells at the end of each subsequent hour. We see that the single bacterial cell leads to over one thousand bacterial cells in just ten hours! And if we were to extrapolate the table to twenty-four hours, we would have over 16 million!

Hour	0	1	2	3	4	5	6	7	8	9	10
Bacteria	1	2	4	8	16	32	64	128	256	512	1024

Table 1

In this chapter, we will explore exponential functions, which can be used for, among other things, modeling growth patterns such as those found in bacteria. We will also investigate logarithmic functions, which are closely related to exponential functions. Both types of functions have numerous real-world applications when it comes to modeling and interpreting data.

16. Todar, PhD, Kenneth. Todar's Online Textbook of Bacteriology. http://textbookofbacteriology.net/growth_3.html.

LEARNING OBJECTIVES

In this section, you will:

- Graph exponential functions.
- Graph exponential functions using transformations.

4.2 GRAPHS OF EXPONENTIAL FUNCTIONS

As we discussed in the previous section, exponential functions are used for many real-world applications such as finance, forensics, computer science, and most of the life sciences. Working with an equation that describes a real-world situation gives us a method for making predictions. Most of the time, however, the equation itself is not enough. We learn a lot about things by seeing their pictorial representations, and that is exactly why graphing exponential equations is a powerful tool. It gives us another layer of insight for predicting future events.

Graphing Exponential Functions

Before we begin graphing, it is helpful to review the behavior of exponential growth. Recall the table of values for a function of the form $f(x) = b^x$ whose base is greater than one. We'll use the function $f(x) = 2^x$. Observe how the output values in **Table 1** change as the input increases by 1.

x	-3	-2	-1	0	1	2	3
$f(x) = 2x$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Table 1

Each output value is the product of the previous output and the base, 2. We call the base 2 the *constant ratio*. In fact, for any exponential function with the form $f(x) = ab^x$, b is the constant ratio of the function. This means that as the input increases by 1, the output value will be the product of the base and the previous output, regardless of the value of a.

Notice from the table that

- the output values are positive for all values of x;
- as x increases, the output values increase without bound; and
- as x decreases, the output values grow smaller, approaching zero.

Figure 1 shows the exponential growth function $f(x) = 2^x$.

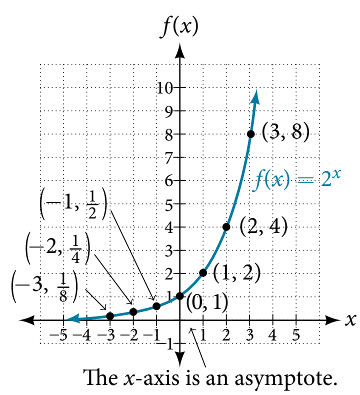


Figure 1 Notice that the graph gets close to the x-axis, but never touches it.

The domain of $f(x) = 2^x$ is all real numbers, the range is $(0, \infty)$, and the horizontal asymptote is $y = 0$.

To get a sense of the behavior of exponential decay, we can create a table of values for a function of the form $f(x) = b^x$ whose base is between zero and one. We'll use the function $g(x) = \left(\frac{1}{2}\right)^x$. Observe how the output values in **Table 2** change as the input increases by 1.

x	-3	-2	-1	0	1	2	3
$g(x) = \left(\frac{1}{2}\right)^x$	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

Table 2

Again, because the input is increasing by 1, each output value is the product of the previous output and the base, or constant ratio $\frac{1}{2}$.

Notice from the table that

- the output values are positive for all values of x;
- as x increases, the output values grow smaller, approaching zero; and
- as x decreases, the output values grow without bound.

Figure 2 shows the exponential decay function, $g(x) = \left(\frac{1}{2}\right)^x$.

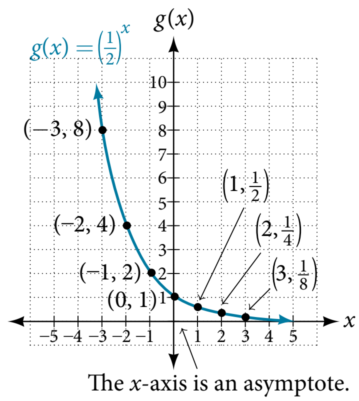


Figure 2

The domain of $g(x) = \left(\frac{1}{2}\right)^x$ is all real numbers, the range is $(0, \infty)$, and the horizontal asymptote is $y = 0$.

characteristics of the graph of the parent function $f(x) = b^x$

An exponential function with the form $f(x) = b^x$, $b > 0$, $b \neq 1$, has these characteristics:

- one-to-one function
- horizontal asymptote: $y = 0$
- domain: $(-\infty, \infty)$
- range: $(0, \infty)$
- x-intercept: none
- y-intercept: $(0, 1)$
- increasing if $b > 1$
- decreasing if $b < 1$

Figure 3 compares the graphs of exponential growth and decay functions.

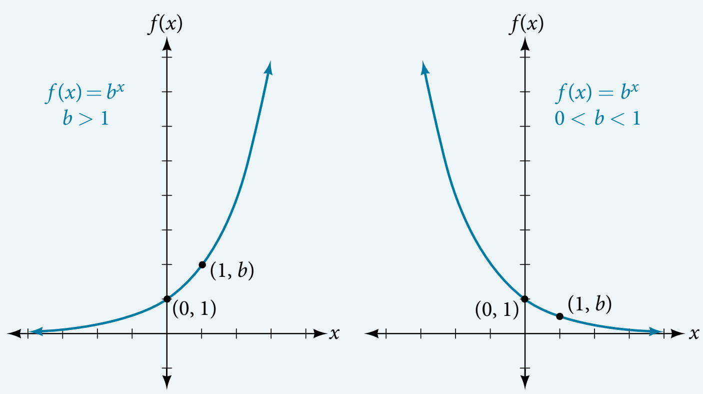


Figure 3

How To...

Given an exponential function of the form $f(x) = b^x$, graph the function.

1. Create a table of points.
2. Plot at least 3 point from the table, including the y-intercept $(0, 1)$.
3. Draw a smooth curve through the points.
4. State the domain, $(-\infty, \infty)$, the range, $(0, \infty)$, and the horizontal asymptote, $y = 0$.

Example 1 **Sketching the Graph of an Exponential Function of the Form $f(x) = b^x$**

Sketch a graph of $f(x) = 0.25^x$. State the domain, range, and asymptote.

Solution Before graphing, identify the behavior and create a table of points for the graph.

- Since $b = 0.25$ is between zero and one, we know the function is decreasing. The left tail of the graph will increase without bound, and the right tail will approach the asymptote $y = 0$.
- Create a table of points as in **Table 3**.

x	-3	-2	-1	0	1	2	3
$f(x) = 0.25^x$	64	16	4	1	0.25	0.0625	0.015625

Table 3

- Plot the y-intercept, $(0, 1)$, along with two other points. We can use $(-1, 4)$ and $(1, 0.25)$.

Draw a smooth curve connecting the points as in **Figure 4**.

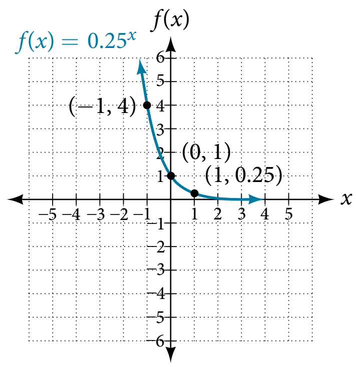


Figure 4

The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.

Try It #1

Sketch the graph of $f(x) = 4^x$. State the domain, range, and asymptote.

Graphing Transformations of Exponential Functions

Transformations of exponential graphs behave similarly to those of other functions. Just as with other parent functions, we can apply the four types of transformations—shifts, reflections, stretches, and compressions—to the parent function $f(x) = b^x$ without loss of shape. For instance, just as the quadratic function maintains its parabolic shape when shifted, reflected, stretched, or compressed, the exponential function also maintains its general shape regardless of the transformations applied.

Graphing a Vertical Shift

The first transformation occurs when we add a constant d to the parent function $f(x) = b^x$, giving us a vertical shift d units in the same direction as the sign. For example, if we begin by graphing a parent function, $f(x) = 2^x$, we can then graph two vertical shifts alongside it, using $d = 3$: the upward shift, $g(x) = 2^x + 3$ and the downward shift, $h(x) = 2^x - 3$. Both vertical shifts are shown in **Figure 5**.

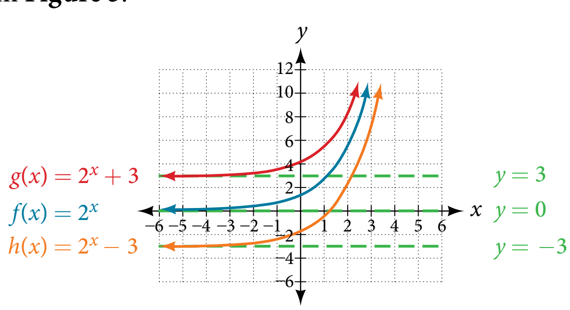


Figure 5

Observe the results of shifting $f(x) = 2^x$ vertically:

- The domain, $(-\infty, \infty)$ remains unchanged.
- When the function is shifted up 3 units to $g(x) = 2^x + 3$:
  - The y-intercept shifts up 3 units to $(0, 4)$.
  - The asymptote shifts up 3 units to $y = 3$.
  - The range becomes $(3, \infty)$.
- When the function is shifted down 3 units to $h(x) = 2^x - 3$:
  - The y-intercept shifts down 3 units to $(0, -2)$.
  - The asymptote also shifts down 3 units to $y = -3$.
  - The range becomes $(-3, \infty)$.

Graphing a Horizontal Shift

The next transformation occurs when we add a constant c to the input of the parent function $f(x) = b^x$, giving us a horizontal shift c units in the *opposite* direction of the sign. For example, if we begin by graphing the parent function $f(x) = 2^x$, we can then graph two horizontal shifts alongside it, using $c = 3$: the shift left, $g(x) = 2^{x+3}$, and the shift right, $h(x) = 2^{x-3}$. Both horizontal shifts are shown in **Figure 6**.

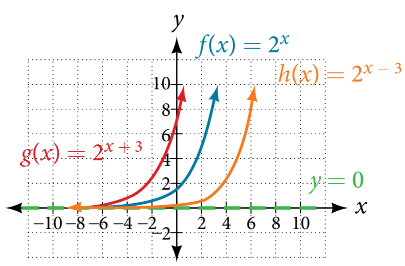


Figure 6

Observe the results of shifting $f(x) = 2^x$ horizontally:

- The domain, $(-\infty, \infty)$, remains unchanged.
- The asymptote, $y = 0$, remains unchanged.
- The y-intercept shifts such that:
  - When the function is shifted left 3 units to $g(x) = 2^{x+3}$, the y-intercept becomes $(0, 8)$. This is because $2^{x+3} = (8)2^x$, so the initial value of the function is 8.
  - When the function is shifted right 3 units to $h(x) = 2^{x-3}$, the y-intercept becomes $\left(0, \frac{1}{8}\right)$. Again, see that $2^{x-3} = \left(\frac{1}{8}\right)2^x$, so the initial value of the function is $\frac{1}{8}$.

shifts of the parent function $f(x) = b^x$

For any constants c and d, the function $f(x) = b^{x+c} + d$ shifts the parent function $f(x) = b^x$

- vertically d units, in the *same* direction of the sign of d.
- horizontally c units, in the *opposite* direction of the sign of c.
- The y-intercept becomes $(0, b^c + d)$.
- The horizontal asymptote becomes $y = d$.
- The range becomes (d, ∞).
- The domain, $(-\infty, \infty)$, remains unchanged.

How To...

Given an exponential function with the form $f(x) = b^{x+c} + d$, graph the translation.

1. Draw the horizontal asymptote $y = d$.
2. Identify the shift as $(-c, d)$. Shift the graph of $f(x) = b^x$ left c units if c is positive, and right c units if c is negative.
3. Shift the graph of $f(x) = b^x$ up d units if d is positive, and down d units if d is negative.
4. State the domain, $(-\infty, \infty)$, the range, (d, ∞), and the horizontal asymptote $y = d$.

Example 2 Graphing a Shift of an Exponential Function

Graph $f(x) = 2^{x+1} - 3$. State the domain, range, and asymptote.

Solution We have an exponential equation of the form $f(x) = b^{x+c} + d$, with $b = 2$, $c = 1$, and $d = -3$.

Draw the horizontal asymptote $y = d$, so draw $y = -3$.

Identify the shift as $(-c, d)$, so the shift is $(-1, -3)$.

Shift the graph of $f(x) = b^x$ left 1 units and down 3 units.

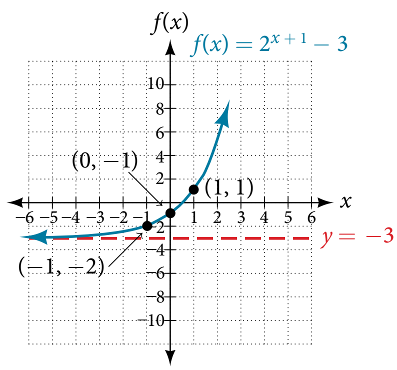


Figure 7

The domain is $(-\infty, \infty)$; the range is $(-3, \infty)$; the horizontal asymptote is $y = -3$.

Try It #2

Graph $f(x) = 2^{x-1} + 3$. State domain, range, and asymptote.

How To...

Given an equation of the form $f(x) = b^{x+c} + d$ for x, use a graphing calculator to approximate the solution.

1. Press **[Y=]**. Enter the given exponential equation in the line headed "$\mathbf{Y_1=}$".
2. Enter the given value for $f(x)$ in the line headed "$\mathbf{Y_2=}$".
3. Press **[WINDOW]**. Adjust the y-axis so that it includes the value entered for "$\mathbf{Y_2=}$".
4. Press **[GRAPH]** to observe the graph of the exponential function along with the line for the specified value of $f(x)$.
5. To find the value of x, we compute the point of intersection. Press **[2ND]** then **[CALC]**. Select "intersect" and press **[ENTER]** three times. The point of intersection gives the value of x for the indicated value of the function.

Example 3 Approximating the Solution of an Exponential Equation

Solve $42 = 1.2(5)^x + 2.8$ graphically. Round to the nearest thousandth.

Solution Press **[Y=]** and enter $1.2(5)^x + 2.8$ next to $\mathbf{Y_1}$=. Then enter 42 next to $\mathbf{Y_2}$=. For a window, use the values -3 to 3 for x and -5 to 55 for y. Press **[GRAPH]**. The graphs should intersect somewhere near $x = 2$.

For a better approximation, press **[2ND]** then **[CALC]**. Select **[5: intersect]** and press **[ENTER]** three times. The x-coordinate of the point of intersection is displayed as 2.1661943. (Your answer may be different if you use a different window or use a different value for **Guess?**) To the nearest thousandth, $x \approx 2.166$.

Try It #3

Solve $4 = 7.85(1.15)^x - 2.27$ graphically. Round to the nearest thousandth.

Graphing a Stretch or Compression

While horizontal and vertical shifts involve adding constants to the input or to the function itself, a stretch or compression occurs when we multiply the parent function $f(x) = b^x$ by a constant $|a| > 0$. For example, if we begin by graphing the parent function $f(x) = 2^x$, we can then graph the stretch, using $a = 3$, to get $g(x) = 3(2)^x$ as shown on the left in **Figure 8**, and the compression, using $a = \frac{1}{3}$, to get $h(x) = \frac{1}{3}(2)^x$ as shown on the right in **Figure 8**.

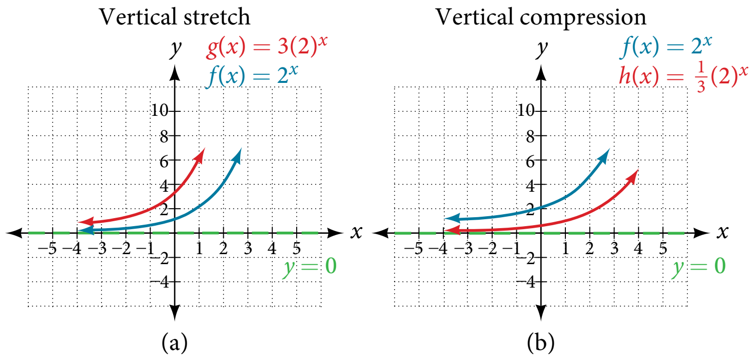


Figure 8 (a) $g(x) = 3(2)^x$ stretches the graph of $f(x) = 2^x$ vertically by a factor of 3. (b) $h(x) = \frac{1}{3}(2)^x$ compresses the graph of $f(x) = 2^x$ vertically by a factor of $\frac{1}{3}$.

stretches and compressions of the parent function $f(x) = b^x$

For any factor $a > 0$, the function $f(x) = a(b)^x$

- is stretched vertically by a factor of a if $|a| > 1$.
- is compressed vertically by a factor of a if $|a| < 1$.
- has a y-intercept of $(0, a)$.
- has a horizontal asymptote at $y = 0$, a range of $(0, \infty)$, and a domain of $(-\infty, \infty)$, which are unchanged from the parent function.

Example 4 Graphing the Stretch of an Exponential Function

Sketch a graph of $f(x) = 4\left(\frac{1}{2}\right)^x$. State the domain, range, and asymptote.

Solution Before graphing, identify the behavior and key points on the graph.

- Since $b = \frac{1}{2}$ is between zero and one, the left tail of the graph will increase without bound as x decreases, and the right tail will approach the x-axis as x increases.
- Since $a = 4$, the graph of $f(x) = \left(\frac{1}{2}\right)^x$ will be stretched by a factor of 4.
- Create a table of points as shown in **Table 4**.

x	-3	-2	-1	0	1	2	3
$f(x) = 4\left(\frac{1}{2}\right)^x$	32	16	8	4	2	1	0.5

Table 4

- Plot the y-intercept, $(0, 4)$, along with two other points. We can use $(-1, 8)$ and $(1, 2)$.

Draw a smooth curve connecting the points, as shown in **Figure 9**.

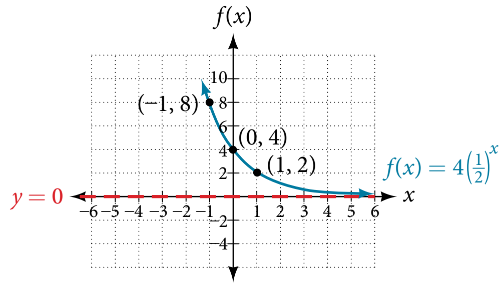


Figure 9

The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.

Try It #4

Sketch the graph of $f(x) = \frac{1}{2}(4)^x$. State the domain, range, and asymptote.

Graphing Reflections

In addition to shifting, compressing, and stretching a graph, we can also reflect it about the x-axis or the y-axis. When we multiply the parent function $f(x) = b^x$ by -1, we get a reflection about the x-axis. When we multiply the input by -1, we get a reflection about the y-axis. For example, if we begin by graphing the parent function $f(x) = 2^x$, we can then graph the two reflections alongside it. The reflection about the x-axis, $g(x) = -2^x$, is shown on the left side of **Figure 10**, and the reflection about the y-axis $h(x) = 2^{-x}$, is shown on the right side of **Figure 10**.

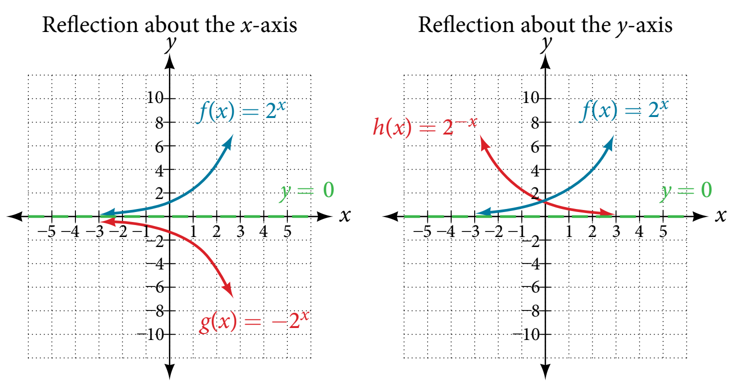


Figure 10 (a) $g(x) = -2^x$ reflects the graph of $f(x) = 2^x$ about the x-axis. (b) $g(x) = 2^{-x}$ reflects the graph of $f(x) = 2^x$ about the y-axis.

reflections of the parent function $f(x) = b^x$

The function $f(x) = -b^x$

- reflects the parent function $f(x) = b^x$ about the x-axis.
- has a y-intercept of $(0, -1)$.
- has a range of $(-\infty, 0)$.
- has a horizontal asymptote at $y = 0$ and domain of $(-\infty, \infty)$, which are unchanged from the parent function.

The function $f(x) = b^{-x}$

- reflects the parent function $f(x) = b^x$ about the y-axis.
- has a y-intercept of $(0, 1)$, a horizontal asymptote at $y = 0$, a range of $(0, \infty)$, and a domain of $(-\infty, \infty)$, which are unchanged from the parent function.

Example 5 Writing and Graphing the Reflection of an Exponential Function

Find and graph the equation for a function, $g(x)$, that reflects $f(x) = \left(\frac{1}{4}\right)^x$ about the x-axis. State its domain, range, and asymptote.

Solution Since we want to reflect the parent function $f(x) = \left(\frac{1}{4}\right)^x$ about the x-axis, we multiply $f(x)$ by -1 to get, $g(x) = -\left(\frac{1}{4}\right)^x$. Next we create a table of points as in **Table 5**.

x	-3	-2	-1	0	1	2	3
$g(x) = -\left(\frac{1}{4}\right)^x$	-64	-16	-4	-1	-0.25	-0.0625	-0.0156

Table 5

Plot the y-intercept, $(0, -1)$, along with two other points. We can use $(-1, -4)$ and $(1, -0.25)$.

Draw a smooth curve connecting the points:

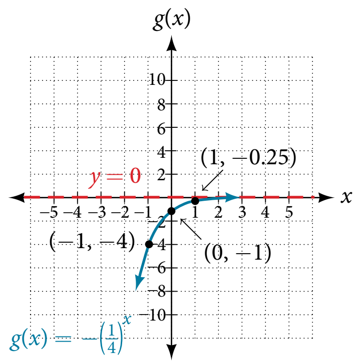


Figure 11

The domain is $(-\infty, \infty)$; the range is $(-\infty, 0)$; the horizontal asymptote is $y = 0$.

Try It #5

Find and graph the equation for a function, $g(x)$, that reflects $f(x) = 1.25^x$ about the y-axis. State its domain, range, and asymptote.

Summarizing Translations of the Exponential Function

Now that we have worked with each type of translation for the exponential function, we can summarize them in **Table 6** to arrive at the general equation for translating exponential functions.

Translations of the Parent Function $f(x) = b^x$	
Translation	**Form**
Shift • Horizontally c units to the left • Vertically d units up	$f(x) = b^{x+c} + d$
Stretch and Compress • Stretch if $\lvert a \rvert > 1$ • Compression if $0 < \lvert a \rvert < 1$	$f(x) = ab^x$
Reflect about the x-axis	$f(x) = -b^x$
Reflect about the y-axis	$f(x) = b^{-x} = \left(\frac{1}{b}\right)^x$
General equation for all translations	$f(x) = ab^{x+c} + d$

Table 6

translations of exponential functions

A translation of an exponential function has the form

$$f(x) = ab^{x+c} + d$$

Where the parent function, $y = b^x$, $b > 1$, is

- shifted horizontally c units to the left.
- stretched vertically by a factor of $|a|$ if $|a| > 0$.
- compressed vertically by a factor of $|a|$ if $0 < |a| < 1$.
- shifted vertically d units.
- reflected about the x-axis when $a < 0$.

Note the order of the shifts, transformations, and reflections follow the order of operations.

Example 6 **Writing a Function from a Description**

Write the equation for the function described below. Give the horizontal asymptote, the domain, and the range.

- $f(x) = e^x$ is vertically stretched by a factor of 2, reflected across the y-axis, and then shifted up 4 units.

Solution We want to find an equation of the general form $f(x) = ab^{x+c} + d$. We use the description provided to find a, b, c, and d.

- We are given the parent function $f(x) = e^x$, so $b = e$.
- The function is stretched by a factor of 2, so $a = 2$.
- The function is reflected about the y-axis. We replace x with $-x$ to get: e^{-x}.
- The graph is shifted vertically 4 units, so $d = 4$.

Substituting in the general form we get,

$$\begin{aligned} f(x) &= ab^{x+c} + d \\ &= 2e^{-x+0} + 4 \\ &= 2e^{-x} + 4 \end{aligned}$$

The domain is $(-\infty, \infty)$; the range is $(4, \infty)$; the horizontal asymptote is $y = 4$.

Try It #6

Write the equation for function described below. Give the horizontal asymptote, the domain, and the range.

- $f(x) = e^x$ is compressed vertically by a factor of $\frac{1}{3}$, reflected across the x-axis and then shifted down 2 units.

Access this online resource for additional instruction and practice with graphing exponential functions.

- Graph Exponential Functions (http://openstaxcollege.org/l/graphexpfunc)

4.2 SECTION EXERCISES

VERBAL

1. What role does the horizontal asymptote of an exponential function play in telling us about the end behavior of the graph?

2. What is the advantage of knowing how to recognize transformations of the graph of a parent function algebraically?

ALGEBRAIC

3. The graph of $f(x) = 3^x$ is reflected about the y-axis and stretched vertically by a factor of 4. What is the equation of the new function, $g(x)$? State its y-intercept, domain, and range.

4. The graph of $f(x) = \left(\frac{1}{2}\right)^{-x}$ is reflected about the y-axis and compressed vertically by a factor of $\frac{1}{5}$. What is the equation of the new function, $g(x)$? State its y-intercept, domain, and range.

5. The graph of $f(x) = 10^x$ is reflected about the x-axis and shifted upward 7 units. What is the equation of the new function, $g(x)$? State its y-intercept, domain, and range.

6. The graph of $f(x) = (1.68)^x$ is shifted right 3 units, stretched vertically by a factor of 2, reflected about the x-axis, and then shifted downward 3 units. What is the equation of the new function, $g(x)$? State its y-intercept (to the nearest thousandth), domain, and range.

7. The graph of $f(x) = -\frac{1}{2}\left(\frac{1}{4}\right)^{x-2} + 4$ is shifted downward 4 units, and then shifted left 2 units, stretched vertically by a factor of 4, and reflected about the x-axis. What is the equation of the new function, $g(x)$? State its y-intercept, domain, and range.

GRAPHICAL

For the following exercises, graph the function and its reflection about the y-axis on the same axes, and give the y-intercept.

8. $f(x) = 3\left(\frac{1}{2}\right)^x$

9. $g(x) = -2(0.25)^x$

10. $h(x) = 6(1.75)^{-x}$

For the following exercises, graph each set of functions on the same axes.

11. $f(x) = 3\left(\frac{1}{4}\right)^x$, $g(x) = 3(2)^x$, and $h(x) = 3(4)^x$

12. $f(x) = \frac{1}{4}(3)^x$, $g(x) = 2(3)^x$, and $h(x) = 4(3)^x$

For the following exercises, match each function with one of the graphs in **Figure 12**.

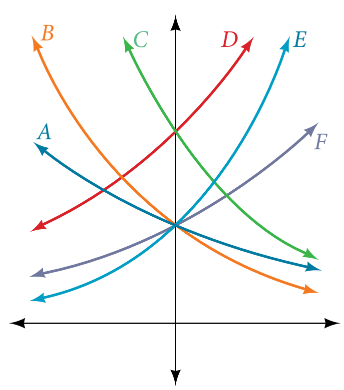


Figure 12

13. $f(x) = 2(0.69)^x$

14. $f(x) = 2(1.28)^x$

15. $f(x) = 2(0.81)^x$

16. $f(x) = 4(1.28)^x$

17. $f(x) = 2(1.59)^x$

18. $f(x) = 4(0.69)^x$

For the following exercises, use the graphs shown in **Figure 13**. All have the form $f(x) = ab^x$.

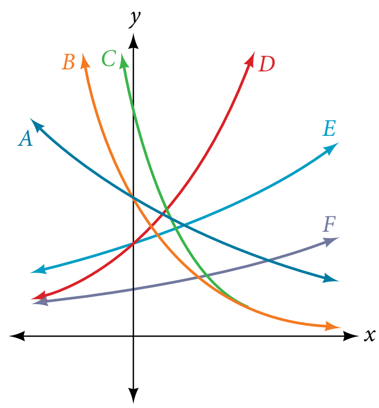


Figure 13

19. Which graph has the largest value for b?

20. Which graph has the smallest value for b?

21. Which graph has the largest value for a?

22. Which graph has the smallest value for a?

For the following exercises, graph the function and its reflection about the x-axis on the same axes.

23. $f(x) = \frac{1}{2}(4)^x$

24. $f(x) = 3(0.75)^x - 1$

25. $f(x) = -4(2)^x + 2$

For the following exercises, graph the transformation of $f(x) = 2^x$. Give the horizontal asymptote, the domain, and the range.

26. $f(x) = 2^{-x}$

27. $h(x) = 2^x + 3$

28. $f(x) = 2^{x-2}$

For the following exercises, describe the end behavior of the graphs of the functions.

29. $f(x) = -5(4)^x - 1$

30. $f(x) = 3\left(\frac{1}{2}\right)^x - 2$

31. $f(x) = 3(4)^{-x} + 2$

For the following exercises, start with the graph of $f(x) = 4^x$. Then write a function that results from the given transformation.

32. Shift $f(x)$ 4 units upward

33. Shift $f(x)$ 3 units downward

34. Shift $f(x)$ 2 units left

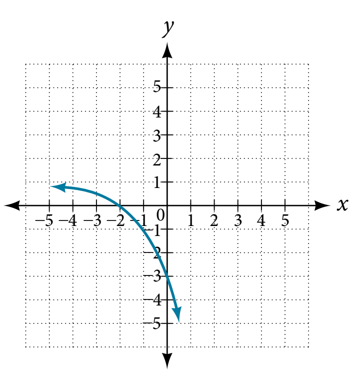
35. Shift $f(x)$ 5 units right

36. Reflect $f(x)$ about the x-axis

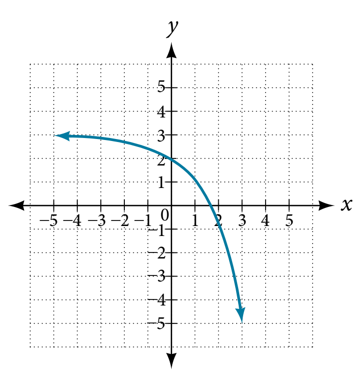
37. Reflect $f(x)$ about the y-axis

For the following exercises, each graph is a transformation of $y = 2^x$. Write an equation describing the transformation.

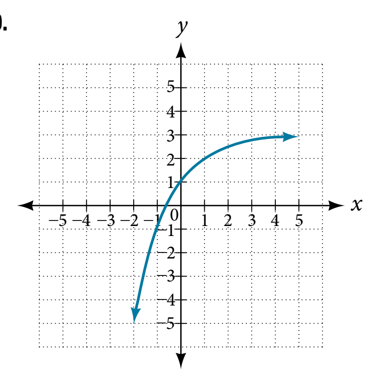
38.



39.

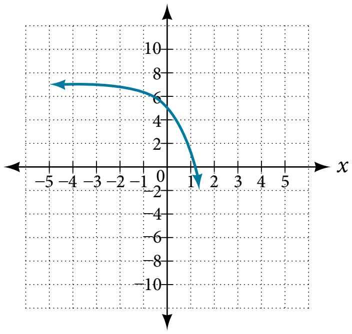


40.

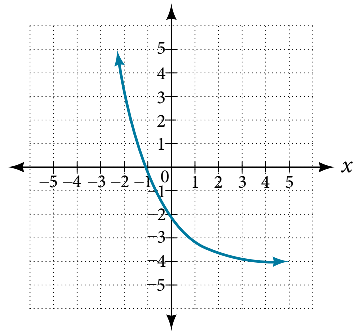


For the following exercises, find an exponential equation for the graph.

41.



42.



NUMERIC

For the following exercises, evaluate the exponential functions for the indicated value of x.

43. $g(x) = \frac{1}{3}(7)^{x-2}$ for $g(6)$.

44. $f(x) = 4(2)^{x-1} - 2$ for $f(5)$.

45. $h(x) = -\frac{1}{2}\left(\frac{1}{2}\right)^x + 6$ for $h(-7)$.

TECHNOLOGY

For the following exercises, use a graphing calculator to approximate the solutions of the equation. Round to the nearest thousandth. $f(x) = ab^x + d$.

46. $-50 = -\left(\frac{1}{2}\right)^{-x}$

47. $116 = \frac{1}{4}\left(\frac{1}{8}\right)^x$

48. $12 = 2(3)^x + 1$

49. $5 = 3\left(\frac{1}{2}\right)^{x-1} - 2$

50. $-30 = -4(2)^{x+2} + 2$

EXTENSIONS

51. Explore and discuss the graphs of $f(x) = (b)^x$ and $g(x) = \left(\frac{1}{b}\right)^x$. Then make a conjecture about the relationship between the graphs of the functions b^x and $\left(\frac{1}{b}\right)^x$ for any real number $b > 0$.

52. Prove the conjecture made in the previous exercise.

53. Explore and discuss the graphs of $f(x) = 4^x$, $g(x) = 4^{x-2}$, and $h(x) = \left(\frac{1}{16}\right)4^x$. Then make a conjecture about the relationship between the graphs of the functions b^x and $\left(\frac{1}{b^n}\right)b^x$ for any real number n and real number $b > 0$.

54. Prove the conjecture made in the previous exercise.