9

Systems of Equations and Inequalities

Figure 1 Enigma machines like this one, once owned by Italian dictator Benito Mussolini, were used by government and military officials for enciphering and deciphering top-secret communications during World War II. (credit: Dave Addey, Flickr)

CHAPTER OUTLINE

Introduction

By 1943, it was obvious to the Nazi regime that defeat was imminent unless it could build a weapon with unlimited destructive power, one that had never been seen before in the history of the world. In September, Adolf Hitler ordered German scientists to begin building an atomic bomb. Rumors and whispers began to spread from across the ocean. Refugees and diplomats told of the experiments happening in Norway. However, Franklin D. Roosevelt wasn't sold, and even doubted British Prime Minister Winston Churchill's warning. Roosevelt wanted undeniable proof. Fortunately, he soon received the proof he wanted when a group of mathematicians cracked the "Enigma" code, proving beyond a doubt that Hitler was building an atomic bomb. The next day, Roosevelt gave the order that the United States begin work on the same.

The Enigma is perhaps the most famous cryptographic device ever known. It stands as an example of the pivotal role cryptography has played in society. Now, technology has moved cryptanalysis to the digital world.

Many ciphers are designed using invertible matrices as the method of message transference, as finding the inverse of a matrix is generally part of the process of decoding. In addition to knowing the matrix and its inverse, the receiver must also know the key that, when used with the matrix inverse, will allow the message to be read.

In this chapter, we will investigate matrices and their inverses, and various ways to use matrices to solve systems of equations. First, however, we will study systems of equations on their own: linear and nonlinear, and then partial fractions. We will not be breaking any secret codes here, but we will lay the foundation for future courses.

LEARNING OBJECTIVES

In this section, you will:

- Solve systems of three equations in three variables.
- Identify inconsistent systems of equations containing three variables.
- Express the solution of a system of dependent equations containing three variables.

9.2 SYSTEMS OF LINEAR EQUATIONS: THREE VARIABLES

Figure 1 (credit: "Elembis," Wikimedia Commons)

John received an inheritance of $12,000 that he divided into three parts and invested in three ways: in a money-market fund paying 3% annual interest; in municipal bonds paying 4% annual interest; and in mutual funds paying 7% annual interest. John invested $4,000 more in municipal funds than in municipal bonds. He earned $670 in interest the first year. How much did John invest in each type of fund?

Understanding the correct approach to setting up problems such as this one makes finding a solution a matter of following a pattern. We will solve this and similar problems involving three equations and three variables in this section. Doing so uses similar techniques as those used to solve systems of two equations in two variables. However, finding solutions to systems of three equations requires a bit more organization and a touch of visual gymnastics.

Solving Systems of Three Equations in Three Variables

In order to solve systems of equations in three variables, known as three-by-three systems, the primary tool we will be using is called Gaussian elimination, named after the prolific German mathematician Karl Friedrich Gauss. While there is no definitive order in which operations are to be performed, there are specific guidelines as to what type of moves can be made. We may number the equations to keep track of the steps we apply. The goal is to eliminate one variable at a time to achieve upper triangular form, the ideal form for a three-by-three system because it allows for straightforward back-substitution to find a solution (x, y, z), which we call an ordered triple. A system in upper triangular form looks like the following:

$$\begin{aligned} Ax + By + Cz &= D \\ Ey + Fz &= G \\ Hz &= K \end{aligned}$$

The third equation can be solved for z, and then we back-substitute to find y and x. To write the system in upper triangular form, we can perform the following operations:

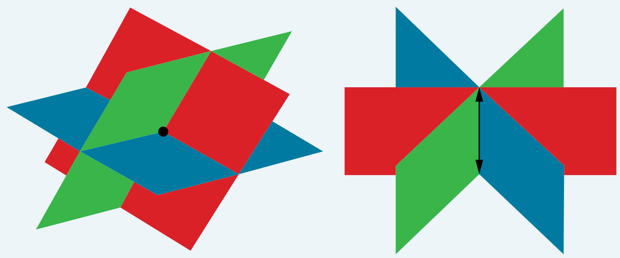
1. Interchange the order of any two equations.
2. Multiply both sides of an equation by a nonzero constant.
3. Add a nonzero multiple of one equation to another equation.

The **solution set** to a three-by-three system is an ordered triple $\{(x, y, z)\}$. Graphically, the ordered triple defines the point that is the intersection of three planes in space. You can visualize such an intersection by imagining any corner in a rectangular room. A corner is defined by three planes: two adjoining walls and the floor (or ceiling). Any point where two walls and the floor meet represents the intersection of three planes.

number of possible solutions

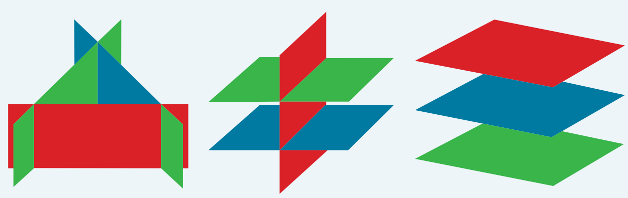
Figure 2 and **Figure 3** illustrate possible solution scenarios for three-by-three systems.

- Systems that have a single solution are those which, after elimination, result in a **solution set** consisting of an ordered triple $\{(x, y, z)\}$. Graphically, the ordered triple defines a point that is the intersection of three planes in space.
- Systems that have an infinite number of solutions are those which, after elimination, result in an expression that is always true, such as $0 = 0$. Graphically, an infinite number of solutions represents a line or coincident plane that serves as the intersection of three planes in space.
- Systems that have no solution are those that, after elimination, result in a statement that is a contradiction, such as $3 = 0$. Graphically, a system with no solution is represented by three planes with no point in common.



(a) (b)

Figure 2 (a)Three planes intersect at a single point, representing a three-by-three system with a single solution. (b) Three planes intersect in a line, representing a three-by-three system with infinite solutions.



(a) (b) (c)

Figure 3 All three figures represent three-by-three systems with no solution. (a) The three planes intersect with each other, but not at a common point. (b) Two of the planes are parallel and intersect with the third plane, but not with each other. (c) All three planes are parallel, so there is no point of intersection.

Example 1 **Determining Whether an Ordered Triple Is a Solution to a System**

Determine whether the ordered triple $(3, -2, 1)$ is a solution to the system.

$$\begin{aligned} x + y + z &= 2 \\ 6x - 4y + 5z &= 31 \\ 5x + 2y + 2z &= 13 \end{aligned}$$

Solution We will check each equation by substituting in the values of the ordered triple for x, y, and z.

$$\begin{aligned} x + y + z &= 2 \\ (3) + (-2) + (1) &= 2 \\ &\text{True} \end{aligned}$$

$$\begin{aligned} 6x - 4y + 5z &= 31 \\ 6(3) - 4(-2) + 5(1) &= 31 \\ 18 + 8 + 5 &= 31 \\ &\text{True} \end{aligned}$$

$$\begin{aligned} 5x + 2y + 2z &= 13 \\ 5(3) + 2(-2) + 2(1) &= 13 \\ 15 - 4 + 2 &= 13 \\ &\text{True} \end{aligned}$$

The ordered triple $(3, -2, 1)$ is indeed a solution to the system.

How To...

Given a linear system of three equations, solve for three unknowns.

1. Pick any pair of equations and solve for one variable.
2. Pick another pair of equations and solve for the same variable.
3. You have created a system of two equations in two unknowns. Solve the resulting two-by-two system.
4. Back-substitute known variables into any one of the original equations and solve for the missing variable.

Example 2 Solving a System of Three Equations in Three Variables by Elimination

Find a solution to the following system:

$$\begin{aligned} x - 2y + 3z &= 9 && (1) \\ -x + 3y - z &= -6 && (2) \\ 2x - 5y + 5z &= 17 && (3) \end{aligned}$$

Solution There will always be several choices as to where to begin, but the most obvious first step here is to eliminate x by adding equations (1) and (2).

$$\begin{array}{rl} x - 2y + 3z = 9 & (1) \\ -x + 3y - z = -6 & (2) \\ \hline y + 2z = 3 & (3) \end{array}$$

The second step is multiplying equation (1) by -2 and adding the result to equation (3). These two steps will eliminate the variable x.

$$\begin{array}{rl} -2x + 4y - 6z = -18 & (1) \text{ multiplied by } -2 \\ 2x - 5y + 5z = 17 & (3) \\ \hline -y - z = -1 & (5) \end{array}$$

In equations (4) and (5), we have created a new two-by-two system. We can solve for z by adding the two equations.

$$\begin{array}{rl} y + 2z = 3 & (4) \\ -y - z = -1 & (5) \\ \hline z = 2 & (6) \end{array}$$

Choosing one equation from each new system, we obtain the upper triangular form:

$$\begin{aligned} x - 2y + 3z &= 9 && (1) \\ y + 2z &= 3 && (4) \\ z &= 2 && (6) \end{aligned}$$

Next, we back-substitute $z = 2$ into equation (4) and solve for y.

$$\begin{aligned} y + 2(2) &= 3 \\ y + 4 &= 3 \\ y &= -1 \end{aligned}$$

Finally, we can back-substitute $z = 2$ and $y = -1$ into equation (1). This will yield the solution for x.

$$\begin{aligned} x - 2(-1) + 3(2) &= 9 \\ x + 2 + 6 &= 9 \\ x &= 1 \end{aligned}$$

The solution is the ordered triple $(1, -1, 2)$. See **Figure 4**.

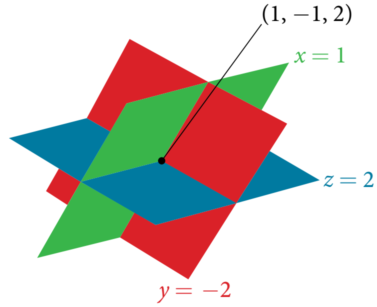


Figure 4

Example 3 Solving a Real-World Problem Using a System of Three Equations in Three Variables

In the problem posed at the beginning of the section, John invested his inheritance of \$12,000 in three different funds: part in a money-market fund paying 3% interest annually; part in municipal bonds paying 4% annually; and the rest in mutual funds paying 7% annually. John invested \$4,000 more in mutual funds than he invested in municipal bonds. The total interest earned in one year was \$670. How much did he invest in each type of fund?

Solution To solve this problem, we use all of the information given and set up three equations. First, we assign a variable to each of the three investment amounts:

$$x = \text{amount invested in money-market fund}$$
$$y = \text{amount invested in municipal bonds}$$
$$z = \text{amount invested in mutual funds}$$

The first equation indicates that the sum of the three principal amounts is \$12,000.

$$x + y + z = 12{,}000$$

We form the second equation according to the information that John invested \$4,000 more in mutual funds than he invested in municipal bonds.

$$z = y + 4{,}000$$

The third equation shows that the total amount of interest earned from each fund equals \$670.

$$0.03x + 0.04y + 0.07z = 670$$

Then, we write the three equations as a system.

$$\begin{aligned} x + y + z &= 12{,}000 \\ -y + z &= 4{,}000 \\ 0.03x + 0.04y + 0.07z &= 670 \end{aligned}$$

To make the calculations simpler, we can multiply the third equation by 100. Thus,

$$\begin{aligned} x + y + z &= 12{,}000 && (1) \\ -y + z &= 4{,}000 && (2) \\ 3x + 4y + 7z &= 67{,}000 && (3) \end{aligned}$$

Step 1. Interchange equation (2) and equation (3) so that the two equations with three variables will line up.

$$\begin{aligned} x + y + z &= 12{,}000 \\ 3x + 4y + 7z &= 67{,}000 \\ -y + z &= 4{,}000 \end{aligned}$$

Step 2. Multiply equation (1) by -3 and add to equation (2). Write the result as row 2.

$$\begin{aligned} x + y + z &= 12{,}000 \\ y + 4z &= 31{,}000 \\ -y + z &= 4{,}000 \end{aligned}$$

Step 3. Add equation (2) to equation (3) and write the result as equation (3).

$$\begin{aligned} x + y + z &= 12{,}000 \\ y + 4z &= 31{,}000 \\ 5z &= 35{,}000 \end{aligned}$$

Step 4. Solve for z in equation (3). Back-substitute that value in equation (2) and solve for y. Then, back-substitute the values for z and y into equation (1) and solve for x.

$$\begin{aligned} 5z &= 35{,}000 \\ z &= 7{,}000 \\ y + 4(7{,}000) &= 31{,}000 \\ y &= 3{,}000 \\ x + 3{,}000 + 7{,}000 &= 12{,}000 \\ x &= 2{,}000 \end{aligned}$$

John invested \$2,000 in a money-market fund, \$3,000 in municipal bonds, and \$7,000 in mutual funds.

Try It #1

Solve the system of equations in three variables.

$$\begin{aligned} 2x + y - 2z &= -1 \\ 3x - 3y - z &= 5 \\ x - 2y + 3z &= 6 \end{aligned}$$

Identifying Inconsistent Systems of Equations Containing Three Variables

Just as with systems of equations in two variables, we may come across an inconsistent system of equations in three variables, which means that it does not have a solution that satisfies all three equations. The equations could represent three parallel planes, two parallel planes and one intersecting plane, or three planes that intersect the other two but not at the same location. The process of elimination will result in a false statement, such as $3 = 7$ or some other contradiction.

Example 4 Solving an Inconsistent System of Three Equations in Three Variables

Solve the following system.

$$\begin{aligned} x - 3y + z &= 4 && (1) \\ -x + 2y - 5z &= 3 && (2) \\ 5x - 13y + 13z &= 8 && (3) \end{aligned}$$

Solution Looking at the coefficients of x, we can see that we can eliminate x by adding equation (1) to equation (2).

$$\begin{aligned} x - 3y + z &= 4 && (1) \\ -x + 2y - 5z &= 3 && (2) \\ \hline -y - 4z &= 7 && (4) \end{aligned}$$

Next, we multiply equation (1) by -5 and add it to equation (3).

$$\begin{aligned} -5x + 15y - 5z &= -20 && (1) \text{ multiplied by } -5 \\ 5x - 13y + 13z &= 8 && (3) \\ \hline 2y + 8z &= -12 && (5) \end{aligned}$$

Then, we multiply equation (4) by 2 and add it to equation (5).

$$\begin{aligned} -2y - 8z &= 14 && (4) \text{ multiplied by } 2 \\ 2y + 8z &= -12 && (5) \\ \hline 0 &= 2 \end{aligned}$$

The final equation $0 = 2$ is a contradiction, so we conclude that the system of equations in inconsistent and, therefore, has no solution.

Analysis *In this system, each plane intersects the other two, but not at the same location. Therefore, the system is inconsistent.*

Try It #2

Solve the system of three equations in three variables.

$$\begin{aligned} x + y + z &= 2 \\ y - 3z &= 1 \\ 2x + y + 5z &= 0 \end{aligned}$$

Expressing the Solution of a System of Dependent Equations Containing Three Variables

We know from working with systems of equations in two variables that a dependent system of equations has an infinite number of solutions. The same is true for dependent systems of equations in three variables. An infinite number of solutions can result from several situations. The three planes could be the same, so that a solution to one equation will be the solution to the other two equations. All three equations could be different but they intersect on a line, which has infinite solutions. Or two of the equations could be the same and intersect the third on a line.

Example 5 **Finding the Solution to a Dependent System of Equations**

Find the solution to the given system of three equations in three variables.

$$\begin{aligned} 2x + y - 3z &= 0 & (1) \\ 4x + 2y - 6z &= 0 & (2) \\ x - y + z &= 0 & (3) \end{aligned}$$

Solution First, we can multiply equation (1) by -2 and add it to equation (2).

$$\begin{array}{rl} -4x - 2y + 6z = 0 & \text{equation (1) multiplied by } -2 \\ 4x + 2y - 6z = 0 & (2) \\ \hline 0 = 0 & \end{array}$$

We do not need to proceed any further. The result we get is an identity, $0 = 0$, which tells us that this system has an infinite number of solutions. There are other ways to begin to solve this system, such as multiplying equation (3) by -2, and adding it to equation (1). We then perform the same steps as above and find the same result, $0 = 0$.

When a system is dependent, we can find general expressions for the solutions. Adding equations (1) and (3), we have

$$\begin{aligned} 2x + y - 3z &= 0 \\ x - y + z &= 0 \\ 3x - 2z &= 0 \end{aligned}$$

We then solve the resulting equation for z.

$$\begin{aligned} 3x - 2z &= 0 \\ z &= \frac{3}{2}x \end{aligned}$$

We back-substitute the expression for z into one of the equations and solve for y.

$$
\begin{aligned}
2x + y - 3\left(\frac{3}{2}x\right) &= 0 \\
2x + y - \frac{9}{2}x &= 0 \\
y &= \frac{9}{2}x - 2x \\
y &= \frac{5}{2}x
\end{aligned}
$$

So the general solution is $\left(x, \frac{5}{2}x, \frac{3}{2}x\right)$. In this solution, x can be any real number. The values of y and z are dependent on the value selected for x.

Analysis *As shown in **Figure 5**, two of the planes are the same and they intersect the third plane on a line. The solution set is infinite, as all points along the intersection line will satisfy all three equations.*

Figure 5

Q & A...

Does the generic solution to a dependent system always have to be written in terms of x?

No, you can write the generic solution in terms of any of the variables, but it is common to write it in terms of x and if needed x and y.

Try It #3

Solve the following system.

$$
\begin{aligned}
x + y + z &= 7 \\
3x - 2y - z &= 4 \\
x + 6y + 5z &= 24
\end{aligned}
$$

Access these online resources for additional instruction and practice with systems of equations in three variables.

- Ex 1: System of Three Equations with Three Unknowns Using Elimination (http://openstaxcollege.org/l/systhree)
- Ex. 2: System of Three Equations with Three Unknowns Using Elimination (http://openstaxcollege.org/l/systhelim)

9.2 SECTION EXERCISES

VERBAL

1. Can a linear system of three equations have exactly two solutions? Explain why or why not

2. If a given ordered triple solves the system of equations, is that solution unique? If so, explain why. If not, give an example where it is not unique.

3. If a given ordered triple does not solve the system of equations, is there no solution? If so, explain why. If not, give an example.

4. Using the method of addition, is there only one way to solve the system?

5. Can you explain whether there can be only one method to solve a linear system of equations? If yes, give an example of such a system of equations. If not, explain why not.

ALGEBRAIC

For the following exercises, determine whether the ordered triple given is the solution to the system of equations.

6. $\begin{aligned} 2x - 6y + 6z &= -12 \\ x + 4y + 5z &= -1 \\ -x + 2y + 3z &= -1 \end{aligned}$ and $(0, 1, -1)$

7. $\begin{aligned} 6x - y + 3z &= 6 \\ 3x + 5y + 2z &= 0 \\ x + y &= 0 \end{aligned}$ and $(3, -3, -5)$

8. $\begin{aligned} 6x - 7y + z &= 2 \\ -x - y + 3z &= 4 \\ 2x + y - z &= 1 \end{aligned}$ and $(4, 2, -6)$

9. $\begin{aligned} x - y &= 0 \\ x - z &= 5 \\ x - y + z &= -1 \end{aligned}$ and $(4, 4, -1)$

10. $\begin{aligned} -x - y + 2z &= 3 \\ 5x + 8y - 3z &= 4 \\ -x + 3y - 5z &= -5 \end{aligned}$ and $(4, 1, -7)$

For the following exercises, solve each system by substitution.

11. $\begin{aligned} 3x - 4y + 2z &= -15 \\ 2x + 4y + z &= 16 \\ 2x + 3y + 5z &= 20 \end{aligned}$

12. $\begin{aligned} 5x - 2y + 3z &= 20 \\ 2x - 4y - 3z &= -9 \\ x + 6y - 8z &= 21 \end{aligned}$

13. $\begin{aligned} 5x + 2y + 4z &= 9 \\ -3x + 2y + z &= 10 \\ 4x - 3y + 5z &= -3 \end{aligned}$

14. $\begin{aligned} 4x - 3y + 5z &= 31 \\ -x + 2y + 4z &= 20 \\ x + 5y - 2z &= -29 \end{aligned}$

15. $\begin{aligned} 5x - 2y + 3z &= 4 \\ -4x + 6y - 7z &= -1 \\ 3x + 2y - z &= 4 \end{aligned}$

16. $\begin{aligned} 4x + 6y + 9z &= 0 \\ -5x + 2y - 6z &= 3 \\ 7x - 4y + 3z &= -3 \end{aligned}$

For the following exercises, solve each system by Gaussian elimination.

17. $\begin{aligned} 2x - y + 3z &= 17 \\ -5x + 4y - 2z &= -46 \\ 2y + 5z &= -7 \end{aligned}$

18. $\begin{aligned} 5x - 6y + 3z &= 50 \\ -x + 4y &= 10 \\ 2x - z &= 10 \end{aligned}$

19. $\begin{aligned} 2x + 3y - 6z &= 1 \\ -4x - 6y + 12z &= -2 \\ x + 2y + 5z &= 10 \end{aligned}$

20. $\begin{aligned} 4x + 6y - 2z &= 8 \\ 6x + 9y - 3z &= 12 \\ -2x - 3y + z &= -4 \end{aligned}$

21. $\begin{aligned} 2x + 3y - 4z &= 5 \\ -3x + 2y + z &= 11 \\ -x + 5y + 3z &= 4 \end{aligned}$

22. $\begin{aligned} 10x + 2y - 14z &= 8 \\ -x - 2y - 4z &= -1 \\ -12x - 6y + 6z &= -12 \end{aligned}$

23. $\begin{aligned} x + y + z &= 14 \\ 2y + 3z &= -14 \\ -16y - 24z &= -112 \end{aligned}$

24. $\begin{aligned} 5x - 3y + 4z &= -1 \\ -4x + 2y - 3z &= 0 \\ -x + 5y + 7z &= -11 \end{aligned}$

25. $\begin{aligned} x + y + z &= 0 \\ 2x - y + 3z &= 0 \\ x - z &= 0 \end{aligned}$

26. $$\begin{aligned} 3x + 2y - 5z &= 6 \\ 5x - 4y + 3z &= -12 \\ 4x + 5y - 2z &= 15 \end{aligned}$$

27. $$\begin{aligned} x + y + z &= 0 \\ 2x - y + 3z &= 0 \\ x - z &= 1 \end{aligned}$$

28. $$\begin{aligned} 3x - \frac{1}{2}y - z &= -\frac{1}{2} \\ 4x + z &= 3 \\ -x + \frac{3}{2}y &= \frac{5}{2} \end{aligned}$$

29. $$\begin{aligned} 6x - 5y + 6z &= 38 \\ \frac{1}{5}x - \frac{1}{2}y + \frac{3}{5}z &= 1 \\ -4x - \frac{3}{2}y - z &= -74 \end{aligned}$$

30. $$\begin{aligned} \frac{1}{2}x - \frac{1}{5}y + \frac{2}{5}z &= -\frac{13}{10} \\ \frac{1}{4}x - \frac{2}{5}y - \frac{1}{5}z &= -\frac{7}{20} \\ -\frac{1}{2}x - \frac{3}{4}y - \frac{1}{2}z &= -\frac{5}{4} \end{aligned}$$

31. $$\begin{aligned} -\frac{1}{3}x - \frac{1}{2}y - \frac{1}{4}z &= \frac{3}{4} \\ -\frac{1}{2}x - \frac{1}{4}y - \frac{1}{2}z &= 2 \\ -\frac{1}{4}x - \frac{3}{4}y - \frac{1}{2}z &= -\frac{1}{2} \end{aligned}$$

32. $$\begin{aligned} \frac{1}{2}x - \frac{1}{4}y + \frac{3}{4}z &= 0 \\ \frac{1}{4}x - \frac{1}{10}y + \frac{2}{5}z &= -2 \\ \frac{1}{8}x + \frac{1}{5}y - \frac{1}{8}z &= 2 \end{aligned}$$

33. $$\begin{aligned} \frac{4}{5}x - \frac{7}{8}y + \frac{1}{2}z &= 1 \\ -\frac{4}{5}x - \frac{3}{4}y + \frac{1}{3}z &= -8 \\ -\frac{2}{5}x - \frac{7}{8}y + \frac{1}{2}z &= -5 \end{aligned}$$

34. $$\begin{aligned} -\frac{1}{3}x - \frac{1}{8}y + \frac{1}{6}z &= -\frac{4}{3} \\ -\frac{2}{3}x - \frac{7}{8}y + \frac{1}{3}z &= -\frac{23}{3} \\ -\frac{1}{3}x - \frac{5}{8}y + \frac{5}{6}z &= 0 \end{aligned}$$

35. $$\begin{aligned} -\frac{1}{4}x - \frac{5}{4}y + \frac{5}{2}z &= -5 \\ -\frac{1}{2}x - \frac{5}{3}y + \frac{5}{4}z &= \frac{55}{12} \\ -\frac{1}{3}x - \frac{1}{3}y + \frac{1}{3}z &= \frac{5}{3} \end{aligned}$$

36. $$\begin{aligned} \frac{1}{40}x + \frac{1}{60}y + \frac{1}{80}z &= \frac{1}{100} \\ -\frac{1}{2}x - \frac{1}{3}y - \frac{1}{4}z &= -\frac{1}{5} \\ \frac{3}{8}x + \frac{3}{12}y + \frac{3}{16}z &= \frac{3}{20} \end{aligned}$$

37. $$\begin{aligned} 0.1x - 0.2y + 0.3z &= 2 \\ 0.5x - 0.1y + 0.4z &= 8 \\ 0.7x - 0.2y + 0.3z &= 8 \end{aligned}$$

38. $$\begin{aligned} 0.2x + 0.1y - 0.3z &= 0.2 \\ 0.8x + 0.4y - 1.2z &= 0.1 \\ 1.6x + 0.8y - 2.4z &= 0.2 \end{aligned}$$

39. $$\begin{aligned} 1.1x + 0.7y - 3.1z &= -1.79 \\ 2.1x + 0.5y - 1.6z &= -0.13 \\ 0.5x + 0.4y - 0.5z &= -0.07 \end{aligned}$$

40. $$\begin{aligned} 0.5x - 0.5y + 0.5z &= 10 \\ 0.2x - 0.2y + 0.2z &= 4 \\ 0.1x - 0.1y + 0.1z &= 2 \end{aligned}$$

41. $$\begin{aligned} 0.1x + 0.2y + 0.3z &= 0.37 \\ 0.1x - 0.2y - 0.3z &= -0.27 \\ 0.5x - 0.1y - 0.3z &= -0.03 \end{aligned}$$

42. $$\begin{aligned} 0.5x - 0.5y - 0.3z &= 0.13 \\ 0.4x - 0.1y - 0.3z &= 0.11 \\ 0.2x - 0.8y - 0.9z &= -0.32 \end{aligned}$$

43. $$\begin{aligned} 0.5x + 0.2y - 0.3z &= 1 \\ 0.4x - 0.6y + 0.7z &= 0.8 \\ 0.3x - 0.1y - 0.9z &= 0.6 \end{aligned}$$

44. $$\begin{aligned} 0.3x + 0.3y + 0.5z &= 0.6 \\ 0.4x + 0.4y + 0.4z &= 1.8 \\ 0.4x + 0.2y + 0.1z &= 1.6 \end{aligned}$$

45. $$\begin{aligned} 0.8x + 0.8y + 0.8z &= 2.4 \\ 0.3x - 0.5y + 0.2z &= 0 \\ 0.1x + 0.2y + 0.3z &= 0.6 \end{aligned}$$

EXTENSIONS

For the following exercises, solve the system for *x*, *y*, and *z*.

46. $$\begin{aligned} x + y + z &= 3 \\ \frac{x-1}{2} + \frac{y-3}{2} + \frac{z+1}{2} &= 0 \\ \frac{x-2}{3} + \frac{y+4}{3} + \frac{z-3}{3} &= \frac{2}{3} \end{aligned}$$

47. $$\begin{aligned} 5x - 3y - \frac{z+1}{2} &= \frac{1}{2} \\ 6x + \frac{y-9}{2} + 2z &= -3 \\ \frac{x+8}{2} - 4y + z &= 4 \end{aligned}$$

48. $$\begin{aligned} \frac{x+4}{7} - \frac{y-1}{6} + \frac{z+2}{3} &= 1 \\ \frac{x-2}{4} + \frac{y+1}{8} - \frac{z+8}{12} &= 0 \\ \frac{x+6}{3} - \frac{y+2}{3} + \frac{z+4}{2} &= 3 \end{aligned}$$

49. $$\begin{aligned} \frac{x-3}{6} + \frac{y+2}{2} - \frac{z-3}{3} &= 2 \\ \frac{x+2}{4} + \frac{y-5}{2} + \frac{z+4}{2} &= 1 \\ \frac{x+6}{2} - \frac{y-3}{2} + z + 1 &= 9 \end{aligned}$$

50. $$\begin{aligned} \frac{x-1}{3} + \frac{y+3}{4} + \frac{z+2}{6} &= 1 \\ 4x + 3y - 2z &= 11 \\ 0.02x + 0.015y - 0.01z &= 0.065 \end{aligned}$$

REAL-WORLD APPLICATIONS

51. Three even numbers sum up to 108. The smaller is half the larger and the middle number is $\frac{3}{4}$ the larger. What are the three numbers?

52. Three numbers sum up to 147. The smallest number is half the middle number, which is half the largest number. What are the three numbers?

53. At a family reunion, there were only blood relatives, consisting of children, parents, and grandparents, in attendance. There were 400 people total. There were twice as many parents as grandparents, and 50 more children than parents. How many children, parents, and grandparents were in attendance?

54. An animal shelter has a total of 350 animals comprised of cats, dogs, and rabbits. If the number of rabbits is 5 less than one-half the number of cats, and there are 20 more cats than dogs, how many of each animal are at the shelter?

55. Your roommate, Sarah, offered to buy groceries for you and your other roommate. The total bill was \$82. She forgot to save the individual receipts but remembered that your groceries were \$0.05 cheaper than half of her groceries, and that your other roommate's groceries were \$2.10 more than your groceries. How much was each of your share of the groceries?

56. Your roommate, John, offered to buy household supplies for you and your other roommate. You live near the border of three states, each of which has a different sales tax. The total amount of money spent was \$100.75. Your supplies were bought with 5% tax, John's with 8% tax, and your third roommate's with 9% sales tax. The total amount of money spent without taxes is \$93.50. If your supplies before tax were \$1 more than half of what your third roommate's supplies were before tax, how much did each of you spend? Give your answer both with and without taxes.

57. Three coworkers work for the same employer. Their jobs are warehouse manager, office manager, and truck driver. The sum of the annual salaries of the warehouse manager and office manager is \$82,000. The office manager makes \$4,000 more than the truck driver annually. The annual salaries of the warehouse manager and the truck driver total \$78,000. What is the annual salary of each of the co-workers?

58. At a carnival, \$2,914.25 in receipts were taken at the end of the day. The cost of a child's ticket was \$20.50, an adult ticket was \$29.75, and a senior citizen ticket was \$15.25. There were twice as many senior citizens as adults in attendance, and 20 more children than senior citizens. How many children, adult, and senior citizen tickets were sold?

59. A local band sells out for their concert. They sell all 1,175 tickets for a total purse of \$28,112.50. The tickets were priced at \$20 for student tickets, \$22.50 for children, and \$29 for adult tickets. If the band sold twice as many adult as children tickets, how many of each type was sold?

60. In a bag, a child has 325 coins worth \$19.50. There were three types of coins: pennies, nickels, and dimes. If the bag contained the same number of nickels as dimes, how many of each type of coin was in the bag?

61. Last year, at Haven's Pond Car Dealership, for a particular model of BMW, Jeep, and Toyota, one could purchase all three cars for a total of \$140,000. This year, due to inflation, the same cars would cost \$151,830. The cost of the BMW increased by 8%, the Jeep by 5%, and the Toyota by 12%. If the price of last year's Jeep was \$7,000 less than the price of last year's BMW, what was the price of each of the three cars last year?

62. A recent college graduate took advantage of his business education and invested in three investments immediately after graduating. He invested \$80,500 into three accounts, one that paid 4% simple interest, one that paid 4% simple interest, one that paid $3\frac{1}{8}\%$ simple interest, and one that paid $2\frac{1}{2}\%$ simple interest. He earned \$2,670 interest at the end of one year. If the amount of the money invested in the second account was four times the amount invested in the third account, how much was invested in each account?

63. You inherit one million dollars. You invest it all in three accounts for one year. The first account pays 3% compounded annually, the second account pays 4% compounded annually, and the third account pays 2% compounded annually. After one year, you earn $34,000 in interest. If you invest four times the money into the account that pays 3% compared to 2%, how much did you invest in each account?

64. You inherit one hundred thousand dollars. You invest it all in three accounts for one year. The first account pays 4% compounded annually, the second account pays 3% compounded annually, and the third account pays 2% compounded annually. After one year, you earn $3,650 in interest. If you invest five times the money in the account that pays 4% compared to 3%, how much did you invest in each account?

65. The top three countries in oil consumption in a certain year are as follows: the United States, Japan, and China. In millions of barrels per day, the three top countries consumed 39.8% of the world's consumed oil. The United States consumed 0.7% more than four times China's consumption. The United States consumed 5% more than triple Japan's consumption. What percent of the world oil consumption did the United States, Japan, and China consume?[28]

66. The top three countries in oil production in the same year are Saudi Arabia, the United States, and Russia. In millions of barrels per day, the top three countries produced 31.4% of the world's produced oil. Saudi Arabia and the United States combined for 22.1% of the world's production, and Saudi Arabia produced 2% more oil than Russia. What percent of the world oil production did Saudi Arabia, the United States, and Russia produce?[29]

67. The top three sources of oil imports for the United States in the same year were Saudi Arabia, Mexico, and Canada. The three top countries accounted for 47% of oil imports. The United States imported 1.8% more from Saudi Arabia than they did from Mexico, and 1.7% more from Saudi Arabia than they did from Canada. What percent of the United States oil imports were from these three countries?[30]

68. The top three oil producers in the United States in a certain year are the Gulf of Mexico, Texas, and Alaska. The three regions were responsible for 64% of the United States oil production. The Gulf of Mexico and Texas combined for 47% of oil production. Texas produced 3% more than Alaska. What percent of United States oil production came from these regions?[31]

69. At one time, in the United States, 398 species of animals were on the endangered species list. The top groups were mammals, birds, and fish, which comprised 55% of the endangered species. Birds accounted for 0.7% more than fish, and fish accounted for 1.5% more than mammals. What percent of the endangered species came from mammals, birds, and fish?

70. Meat consumption in the United States can be broken into three categories: red meat, poultry, and fish. If fish makes up 4% less than one-quarter of poultry consumption, and red meat consumption is 18.2% higher than poultry consumption, what are the percentages of meat consumption?[32]

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